

*Final Report*

*49 p.*

UNPUB

NASw-3

N64-25659

*Code-1*

*Cat. 13*

NASA CR-55273

# HEAT TRANSFER LABORATORY

PRANDTL NUMBER MEASUREMENTS AND  
THERMAL CONDUCTIVITY, VISCOSITY PREDICTIONS  
FOR AIR, HELIUM, AND AIR-HELIUM MIXTURES

by

W. E. Ibele, J. L. Novotny, and E. R. G. Eckert

December 1963

UNIVERSITY OF MINNESOTA  
INSTITUTE OF TECHNOLOGY  
MECHANICAL ENGINEERING DEPARTMENT

MINNEAPOLIS, MINNESOTA 55455



OTS PRICE

XEROX

MICROFILM

\$

\$

*4.60 ph*

*none*

PRANDTL NUMBER MEASUREMENTS AND  
THERMAL CONDUCTIVITY, VISCOSITY PREDICTIONS  
FOR AIR, HELIUM, AND AIR-HELIUM MIXTURES

by

W. E. Ibele, J. L. Novotny, and E. R. G. Eckert

Summary

Prandtl number measurements of air, helium, and air-helium mixtures have been performed over a temperature range from 300 to 700 K. Various methods of predicting the thermal conductivity and viscosity of binary mixtures were also investigated and results obtained by use of these methods were compared with the experimental results. The method set forth by Hirschfelder, Curtiss, and Bird which uses the Lennard-Jones intermolecular potential was selected and force constants determined from experimental data were applied for predicting thermal conductivity and viscosity. Dimensionless conductivities, viscosities, and Prandtl numbers were calculated over the temperature range of 200 to 3,000 K for mixtures ranging from pure air to pure helium. *Author*

INTRODUCTION

Of the various transport properties of gases, the experimental determination of the coefficient of thermal conductivity encounters the most severe difficulties and accounts for the

present limited knowledge of this property. The methods usually employed for such measurements rest upon Fourier's equation to describe the heat flow through a stagnant gas sample. Unfortunately, the low value for the coefficient of thermal conductivity for gases makes it extremely difficult to hold the heat losses to desirably small values. At higher temperatures, the radiative energy exchange between surfaces of the test cell is difficult to account for with desired accuracy. In addition, there is the requirement that natural convection currents should be avoided if reliable values are to be obtained.

These sources of error are reduced significantly when coefficients of thermal conductivity are derived from direct Prandtl number measurements as proposed by Eckert and Irvine (ref. 1). The large convective heat-transfer coefficients dominate the radiation fields and permit accurate measurements of the Prandtl number. Thermal conductivity coefficients are then obtained by using viscosity and heat-capacity data, properties which are known with greater confidence than the coefficient of thermal conductivity.

A continuing problem in heat transfer calculations is the determination of property information in a range of temperature where experimental measurements are scant or nonexistent. The hazards of extrapolating experimental data to higher temperatures are obvious. Unfortunately, the various

molecular models used to represent the behavior of matter are presently inadequate since the fundamental laws governing molecular interaction are not sufficiently understood to permit the formulation of an integral, independent approach to the determination of property values.

An approach meeting with success is one embracing experiment and theory in a semitheoretical manner. Briefly, the method consists of determining from experimental data, values of the adjustable constants in the intermolecular potential which describes the action between molecules. When the resulting predictions of transport properties agree closely with experiment, extrapolation to regions where experimental data are meager or nonexistent may be undertaken with reasonable confidence.

The gas mixture of air and helium was selected in the present investigation for two reasons. There exists in general a need for measurements and predictions of thermal conductivity for binary gas mixtures at high temperatures and the mixture of a light gas such as helium with air offered an opportunity to study the characteristics of a dissimilar combination. In addition, there is an active interest in the physical properties of such a mixture because of the use of a light gas such as helium in transpiration cooling techniques.

This investigation was conducted at the Heat Transfer Laboratory of the Mechanical Engineering Department of the

University of Minnesota under the sponsorship and with the financial support of the National Aeronautics and Space Administration and the National Science Foundation. Their interest and assistance is gratefully acknowledged. The authors express their appreciation to Mr. Gangu Hingorani who programmed with care the lengthy calculations necessary for the statistical predictions of transport properties and to Mr. David Briggs who assisted with the experimentation.

#### SYMBOLS

$A^*, B^*$	function of $T^*$
$C_p$	heat capacity at constant pressure
$E$	thermal conductivity ratio, experimental over monatomic prediction
$k$	Boltzmann constant; also, thermal conductivity
$M$	molecular weight
$Pr$	Prandtl number
$R$	gas constant, cal/(gm-mole)(°K) or Btu/(lb-mole)(°R)
$T$	absolute temperature or temperature, deg. K
$T^* = k/\epsilon$	
$V$	velocity
$x$	mole fraction
$\epsilon$	depth of Lennard-Jones intermolecular potential
$\epsilon_{12}$	potential minimum for unlike interactions, given empirically by $(\epsilon_1 \epsilon_2)^{1/2}$
$\eta$	dynamic viscosity

$\sigma$	low-energy collision diameter of Lennard-Jones intermolecular potential
$\sigma_{12}$	low-energy collision diameter for unlike molecules, $(\sigma_1 + \sigma_2)/2$
$\Omega(2,2)^*$	collision integral

Subscripts:

mix	of the mixture
mon	molecules assumed to be effectively monatomic
o	value at 273.16 K
r	adiabatic recovery
T	total
1,2	gases 1 and 2, respectively
12	unlike pair of molecules

#### EXPERIMENTAL PROGRAM

An experimental program of Prandtl number measurements on air, helium, and air-helium mixtures over a temperature range of 300 to 700 K and one atmosphere pressure has been accomplished. The experimental procedure employed the method described in reference 1, which is based on the fact that a well-established, unique relation exists between the Prandtl number and the flat-plate recovery factor for laminar, high-velocity, boundary-layer flow. The equipment for making such measurements is discussed in detail in this reference and only a brief description of it will be given here except for such modifications as were necessary to transform the apparatus for work on mixtures.

The experimental apparatus consists of a subsonic nozzle through which a thermocouple wire is suspended along the nozzle axis and parallel to the flow direction. This wire is a butt-welded, differential thermocouple with one junction located upstream of the nozzle where the velocity is low and the other junction located at the nozzle exit plane where the velocity is a maximum. Under the proper conditions, the upstream junction comes into equilibrium with the total temperature of the gas stream and the downstream junction assumes the adiabatic recovery temperature associated with the gas stream at the exit of the nozzle.

In this installation, the thermocouple wire takes the place of a flat plate over which is moving a high velocity gas stream. The voltage reading of the differential thermocouple is proportional to the difference between the total temperature  $T_T$  and the adiabatic recovery temperature  $T_R$ . This reading, together with an independent measurement of the total temperature and the gas velocity at the nozzle exit, is sufficient to specify the recovery factor and the Prandtl number according to the relation

$$\text{Recovery factor} = \text{Pr}^{1/2} = 1 - \frac{2C_p(T_T - T_R)}{V^2}$$

The velocity in the above equation was determined by measuring the static pressure drop across the nozzle, expressing it as a ratio, and using the gas laws for isentropic expansion. The

pressure drop across the nozzle was adjusted to give a Mach number at nozzle exit of about 0.90 to meet the high-velocity condition. The approach section to the nozzle was designed with care to insure that flow over the differential thermocouple wire was laminar in order that this requirement of the above equation was met. Using this arrangement, previous measurements have been reported (ref. 1) of Prandtl number and thermal conductivity values for air at atmospheric pressure, over a temperature range of 60 to 350 F.

Two modifications were made in using the apparatus to measure the Prandtl number of air-helium mixtures. One concerned the differential thermocouple and the other, the addition of tanks for the preparation of known mixtures of the two gases.

A careful analysis of possible systematic errors in these measurements reveals that a deviation from the true Prandtl number value may arise from the fact that a rotationally symmetrical body, that is, the thermocouple wire, is used instead of the two-dimensional plate for which the rule based on the square root of the Prandtl number was derived. This can cause a systematic error which depends upon the ratio of boundary-layer thickness to wire diameter. This error may become serious when one deals with lightweight gases such as helium which have characteristically thick boundary layers and, accordingly, it was necessary to use a thermocouple wire with a larger diameter than that used in the original apparatus.



Increasing the wire diameter, however, increases the heat conduction between the differential thermocouple junctions and may introduce another error into the procedure. To decrease both curvature and conduction errors, a differential thermocouple was made in the shape of a thin-walled (0.004 inch) tube, 0.125 inch in diameter, from Manganin and constantan metals. This thermocouple was used for measuring the air-helium mixture Prandtl numbers at room temperature ( $270 \pm 10$  K) reported in reference 43. Subsequent use of this thermocouple at elevated temperatures encountered erratic voltage outputs, due apparently to changes in the state of stress of the thin walled tube. Attempts to stress relieve the thermocouple and maintain it in a constant stress environment were unsuccessful and led to a return to the solid differential thermocouple (constantan-Manganin, 0.018 inch diameter) described in reference 1. For the reasons given above, the thermocouple diameter was increased to 0.024 inch diameter and the short center section of wire changed from Manganin to chromel because of Manganin deterioration with extended use at high temperature. Chromel was selected to reduce the conduction between junctions, its thermal conductivity being only 60 percent that of Manganin. Results for carbon-dioxide, carbon-dioxide-air mixtures, and helium reported in references 44 and 45, respectively, employed this particular differential thermocouple. Following these measurements deterioration of the constantan sections were

noted and were replaced by alumel. As noted in reference 46, the chromel-alumel pair exhibits an e.m.f. output depending upon its previous temperature history. The thermocouple was first calibrated, then aged for approximately 200 hours at a temperature of 900 to 1,000 F and recalibrated. While a shift in the e.m.f. vs. temperature curve was noted, the slopes used agreed to within 0.5 percent over the temperature range of experimentation. Measurements were made for the pure gases: argon, carbon-dioxide, helium, nitrogen and the mixtures: carbon-dioxide nitrogen, air-helium. Reports on these results are being prepared for publication.

In order to measure accurately the ratios of the air-helium mixtures, a separate mixing tank was used. The constituent gases were introduced into this tank from standard metal bottles containing air and helium. The mixture ratios were determined by making pressure and temperature measurements in both the metal bottles and the mixing tank. In this way it was possible, by using state equations with compressibility corrections, to obtain two independent measurements of the mixture ratios. These measurements agreed within 0.5 percent.

An analysis was made of the possible random and systematic errors in both the mixture-ratio and recovery-factor measurements. For the latter, instrument readings, calibration errors, radiation and conduction effects on the adiabatic condition, and the influence of the cylindrical geometry were considered. It was

predicted from this analysis that the systematic error in the Prandtl number was less than 1.4 percent and the random error, less than  $\pm 1.0$  percent. In the case of the mixture-ratio determinations, a similar analysis predicted negligible systematic errors and random errors no greater than 1 percent.

Two possible errors deserve a separate discussion since they are unique to measurements of gases where the ratio of the molecular weights is significantly greater than unity. These concern the effect of thermal diffusion in the boundary layer on the rule for the square root of the Prandtl number and the variation of physical properties in this layer. It may be recalled that the above relation of Prandtl number and recovery factor was obtained from a solution of the boundary-layer equations under the condition of constant physical properties and in the absence of thermal diffusion (ref. 2). In order to investigate the present conditions, the boundary-layer equations were again solved, taking into account thermal diffusion and physical property dependence upon both temperature and mass concentration (ref. 3). These calculations indicated that the square root of the Prandtl number evaluated at the free-stream mass concentration and the boundary-layer reference temperature used in reference 1 is an accurate representation of the recovery factor within 0.8 percent which amounts to 1.6 percent in the Prandtl number.

Using the techniques described above, measurements were made of the Prandtl number of air over the temperature range 270 to 700 K, the temperature capability of the present device.

The measurements are shown in figure 1 and will be discussed in greater detail in a later section. Numerical data from the smooth curve drawn through the experimental data in figure 1 are given in table 1.

#### ANALYTICAL PROGRAM

##### Calculation Methods for Transport Properties of Binary Mixtures and Pure Gases

Thermal conductivity for binary mixtures.- Various analytical methods have been proposed to determine the thermal conductivity of binary gas mixtures. None of these are completely rigorous for mixtures involving polyatomic gases, however, and agreement with existing experimental data is usually obtained by adjusting the constants that appear in the equation.

Lindsay and Bromley (ref. 4) modified an equation, first proposed by Wassiljewa (ref. 5), based on kinetic theory. Comparisons of calculated and experimental values of conductivity for a large number of gas mixtures were accomplished; the range of discrepancy was from -7.1 to 10 percent.

Enskog derived an expression for calculating the thermal conductivity of a binary gas mixture, an example of which is given in reference 6. It too was developed from the kinetic theory and contains molecular parameters that are evaluated from existing experimental data, usually viscosity measurements. Keyes (ref. 6) found that conductivity values for the binary

mixture of nitrogen and carbon dioxide in the temperature range 0 to 150 C compared favorably with the experimental data when calculated using the Enskog relation. The calculations at 0 C for the mixture of hydrogen and carbon dioxide, however, did not agree satisfactorily with experiment.

Hirschfelder and coworkers (ref. 7) proposed that the conductivity of a binary gas mixture could be determined reliably by

$$k_{\text{mix}} = k_{\text{mon}}(x_1 E_1 + x_2 E_2) \quad (1)$$

where  $k_{\text{mon}}$  is the calculated thermal conductivity of the mixture when the molecules are assumed to be effectively monatomic and of the same molecular weight as the polyatomic molecules involved. The mole fractions of the two components are represented by  $x_1$  and  $x_2$ . The quantity  $E_1 = (k_i)_{\text{exp}} / (k_i)_1$  where  $(k_i)_{\text{exp}}$  represents the experimentally determined conductivity of the pure gas and  $(k_i)_1$ , the approximation to the conductivity of the pure gas, assuming it to be monatomic.

The necessary equations for determining the thermal conductivity of a binary gas mixture according to the method proposed in reference 7 are as follows:

$$\frac{1}{k_{\text{mon}}} = \frac{x_k + Y_k}{1 + Z_k} \quad (2)$$

$$x_k = \frac{x_1^2}{(k_1)_1} + \frac{2x_1x_2}{(k_{12})_1} + \frac{x_2^2}{(k_2)_1} \quad (2a)$$

$$Y_k = \frac{x_1^2}{(k_1)_1} U^{(1)} + \frac{2x_1x_2}{(k_{12})_1} U^{(Y)} + \frac{x_2^2}{(k_2)_1} U^{(2)} \quad (2b)$$

$$Z_k = x_1^2 U^{(1)} + 2x_1x_2 U^{(Z)} + x_2^2 U^{(2)} \quad (2c)$$

$$U^{(1)} = \frac{4}{15} A_{12}^* - \frac{1}{12} \left( \frac{12}{5} B_{12}^* + 1 \right) \frac{M_1}{M_2} + \frac{1}{2} \frac{(M_1 - M_2)^2}{M_1 M_2} \quad (2d)$$

$$U^{(2)} = \frac{4}{15} A_{12}^* - \frac{1}{12} \left( \frac{12}{5} B_{12}^* + 1 \right) \frac{M_2}{M_1} + \frac{1}{2} \frac{(M_2 - M_1)^2}{M_1 M_2} \quad (2e)$$

$$U^{(Y)} = \frac{4}{15} A_{12}^* \left[ \frac{(M_1 + M_2)^2}{4M_1 M_2} \right] \frac{(k_{12})_1^2}{(k_1)_1 (k_2)_1} - \frac{1}{12} \left( \frac{12}{5} B_{12}^* + 1 \right) - \frac{5}{32A_{12}^*} \left( \frac{12}{5} B_{12}^* - 5 \right) \frac{(M_1 - M_2)^2}{M_1 M_2} \quad (2f)$$

$$U^{(Z)} = \frac{4}{15} A_{12}^* \left\{ \left[ \frac{(M_1 + M_2)^2}{4M_1 M_2} \right] \left[ \frac{(k_{12})_1}{(k_1)_1} + \frac{(k_{12})_1}{(k_2)_1} \right] - 1 \right\} - \frac{1}{12} \left( \frac{12}{5} B_{12}^* + 1 \right) \quad (2g)$$

where

$x_1, x_2$  mole fractions of gases 1 and 2

$M_1, M_2$  molecular weights of gases 1 and 2

$A_{12}^*, B_{12}^*$  functions of  $T_{12}^* = kT/\epsilon_{12}$  (ref. 7, p. 1128)

$\epsilon_{12}$  minimum potential energy for interaction of unlike pairs of molecules,  $\sqrt{\epsilon_1 \epsilon_2}$

T temperature, deg. K

$(k_1)_1, (k_2)_1$  first approximation to thermal conductivity of pure gases 1 and 2, cal/cm sec  $^{\circ}\text{K}$

The quantity  $k_{12}$  is given by

$$(k_{12})_1 \times 10^7 = 1989.1 \frac{\sqrt{T(M_1 + M_2)/2M_1M_2}}{\sigma_{12}^2 \Omega_{12}^{(2,2)*}(T_{12}^*)} \quad (2h)$$

and may be regarded as the thermal conductivity of a hypothetical monatomic pure substance, the molecules of which have a molecular weight of  $2M_1M_2/(M_1 + M_2)$  and interact according to a potential curve characterized by the interaction parameters  $\sigma_{12}$  and  $\epsilon_{12}$ . The quantity  $\sigma_{12}$  is the distance in Angstroms at which low-energy molecules collide and is customarily taken as the arithmetical mean of  $\sigma_1$  and  $\sigma_2$ . The expression  $\Omega_{12}^{(2,2)*}$  is the collision integral and indicates the deviation of any particular molecular model from the idealized rigid-sphere model. It is listed for various values of  $T^* = kT/\epsilon$  in reference 7.

Thermal conductivity for pure gases.— The quantities  $(k_1)_1$  and  $(k_2)_1$  are determined in cal/cm sec  $^{\circ}\text{K}$  by

$$(k)_1 \times 10^7 = 1989.1 \frac{\sqrt{T/M}}{\sigma^2 \Omega^{(2,2)*}(T^*)} \quad (2i)$$

where

- M            molecular weight of pure component
- $\sigma$            low-energy collision diameter for interaction of like molecules
- $\Omega(2,2)^*$    collision integral for like molecules, tabulated as functions of  $T^* = kT/\epsilon$  (ref. 7, p. 1126)

It is apparent from the preceding equations that calculations of thermal conductivity employing the Hirschfelder method are lengthy. It was found, however, that better agreement with the experimental data was obtained through this approach. Thus Lindahl (ref. 47) examined the thermal conductivity of eighteen gas pairs over temperatures ranging from 0 to 800 C and verified the superior accuracy of the Hirschfelder method, as compared to the previous methods (refs. 4, 6) and a later scheme by Brokaw (ref. 49).

The molecular parameters used in the Chapman-Enskog theory as developed by Hirschfelder, Curtiss, and Bird are  $\sigma$ , which is a characteristic distance of the interaction potential energy between two molecules (the average collision diameter of simple kinetic theory), and  $\epsilon/k$ , which further characterizes the potential by specifying the depth of the minimum in the potential energy of interaction. Since viscosity data are more abundant and usually more accurate than thermal conductivity data, practice has been to use the potential parameters determined from viscosity data in calculating both viscosity and thermal conductivity. Values of both these parameters for various gases are given in reference 7.



The existence of a large body of experimental measurements of air thermal conductivity, however, suggested that these measurements be analyzed to determine a best pair of force constants to be used to predict thermal-conductivity coefficients. This analysis was accomplished by first dividing the experimental conductivities by the Eucken correction (ref. 48) which accounts approximately for the transfer of energy between translation and the internal degrees of freedom in molecules having two or more atoms. The results of this operation were then analyzed using equation (21). Successive values of  $\epsilon/k$  were chosen; for each choice a set of  $\sigma$  values results, one for each temperature at which experimental values of  $k$  are reported. The  $\epsilon/k$  associated with the set of  $\sigma$  values exhibiting the least-square percent deviation from the average was then selected together with the average  $\sigma$  as the parameters best characterizing the gas behavior with respect to thermal conductivity. The predicted variation of thermal conductivity with temperature for air is shown in figure 2. The experimental air data are given in refs. 9 to 16, 31 and 52. The predicted curve using force constants based on thermal conductivity gives a good fit of the experimental data. This is particularly true at lower temperatures where the data would be expected to be most reliable. At higher temperatures departure from the experimental data occurs.

The experimental data of Stops (ref. 12) extends beyond 1,000 K but involve radiation corrections so large that these were not plotted. The average deviation between statistical predictions and experimental data is about 1 percent; the maximum deviation is 12 percent at 1,200 K.

Figure 3 shows similar comparisons for the thermal conductivity of helium, the experimental data being that reported in references 17 to 20. Statistical predictions based on thermal-conductivity data clearly deviate least from the experimental data. The average deviation is less than 1 percent, the maximum less than 2 percent. Because of the relative success of using thermal-conductivity-determined force constants, it was decided to employ these in determining the thermal conductivities of air-helium mixtures. The force constants determined from the analysis of conductivity data and used for mixture calculations are given in table 2.

Viscosity for binary mixtures.- The methods for predicting the viscosity of binary gas mixtures were reviewed to determine the basis and accuracy of available methods.

A general form of the viscosity for binary gas mixtures was developed by Buddenberg and Wilke (ref. 21) and later simplified by Wilke (ref. 22) by means of the kinetic theory of gases. This method has been used with success in predicting the viscosity of such binary mixtures as hydrogen-air and air-helium (ref. 23). Pulkrabek (ref. 50) examined viscosity

measurements for binary mixtures involving helium as one component however, and verified the superiority of the Hirschfelder method (ref. 7).

According to this method as proposed in reference 7, the viscosity of a binary gas mixture is given by:

$$\frac{1}{(\eta_{\text{mix}})_1} = \frac{X_\eta + Y_\eta}{1 + Z_\eta} \quad (3)$$

where

$$X_\eta = \frac{x_1^2}{(\eta_1)_1} + \frac{2x_1x_2}{(\eta_{12})_1} + \frac{x_2^2}{(\eta_2)_1} \quad (3a)$$

$$Y_\eta = \frac{3}{5} A_{12} * \left\{ \frac{x_1^2}{(\eta_1)_1} \left( \frac{M_1}{M_2} \right) + \frac{2x_1x_2}{(\eta_{12})_1} \left[ \frac{(M_1 + M_2)^2}{4M_1M_2} \right] \frac{(\eta_{12})_1^2}{(\eta_1)_1(\eta_2)_1} + \frac{x_2^2}{(\eta_2)_1} \left( \frac{M_2}{M_1} \right) \right\} \quad (3b)$$

$$Z_\eta = \frac{3}{5} A_{12} * \left( x_1^2 \left( \frac{M_1}{M_2} \right) + 2x_1x_2 \left\{ \left[ \frac{(M_1 + M_2)^2}{4M_1M_2} \right] \left[ \frac{(\eta_{12})_1}{(\eta_2)_1} \right. \right. \right. \right. \\ \left. \left. \left. + \frac{(\eta_{12})_1}{(\eta_2)_1} \right] - 1 \right\} + x_2^2 \left( \frac{M_2}{M_1} \right) \right) \quad (3c)$$

where  $(\eta_1)_1$  and  $(\eta_2)_1$  are the first approximations to the viscosity of the pure gases 1 and 2 in gm/cm sec.

The quantity  $(\eta_{12})_1$  is given by

$$(\eta_{12})_1 \times 10^7 = 266.93 \frac{\sqrt{2M_1M_2T/(M_1 + M_2)}}{\sigma_{12}^2 \Omega_{12}^{(2,2)*}(T_{12}^*)} \quad (3d)$$

and may be regarded as the viscosity of a hypothetical pure substance which has a molecular weight of  $2M_1M_2/(M_1 + M_2)$ . The molecules of this gas interact according to a potential-energy curve described by the molecular parameters  $\sigma_{12}$  and  $\epsilon_{12}$ . The former is the low-energy collision diameter for unlike molecules customarily taken as  $(\sigma_1 + \sigma_2)/2$ ; the latter is the potential minimum for unlike interactions and is given empirically by  $\sqrt{\epsilon_1 \epsilon_2}$ . The quantity  $\Omega_{12}^{(2,2)*}$  is the collision integral and represents the deviation of any particular molecular model from the idealized rigid sphere model. Values are reported in reference 7 with  $T^* = kT/\epsilon$  as argument.

Viscosity for pure gases.- The quantities  $(\eta_1)_1$  and  $(\eta_2)_1$  are given in gm/cm sec by

$$(\eta_1)_1 \times 10^7 = 266.93 \frac{\sqrt{MT}}{\sigma^2 \Omega^{(2,2)*}(T^*)} \quad (3e)$$

where

- |                   |  |
|-------------------|--|
| M                 | molecular weight of pure component                                 |
| $\sigma$          | low-energy collision diameter for interaction of like molecules, A |
| $\Omega^{(2,2)*}$ | collision integral for like molecules, tabulated in reference 7    |

Because this method was consistent with the thermal-conductivity calculations, gave satisfactory results for the

viscosity of pure substances and best results for binary gas mixtures involving helium, it was decided to utilize this approach for determining air-helium viscosities.

Figure 4 presents the statistical predictions of viscosity for air and the results of several experimental investigations (refs. 24 to 27). The values of the National Bureau of Standards tabulated in reference 8 are also shown for comparison. Agreement is excellent up to 1,000 K where the statistical predictions deviate from both experimental data and the NBS best fit. The average deviation of the statistical predictions from experimental data is about 1 percent and the maximum about 2 percent.

Figure 5 presents the results of viscosity predictions for helium compared with experimental measurements (refs. 26 and 28 to 35). Agreement with experimental data for helium was not quite as good as that for air, (deviations: average 1.5 percent, maximum 2.5 percent). These were not considered to be excessive in view of the fact that a common method was employed for predicting the properties of the mixture components. A somewhat better fit of the experimental viscosity measurements for helium could be obtained by selecting a pair of force constants other than those given in reference 7 and repeated in table 2; however, these were found to be quite satisfactory in predicting viscosities of other binary mixtures

involving helium. Thus figure 6 shows the measured viscosities of three different binary mixtures having helium as one component (refs. 28, 34, 36, and 37). Also shown are the predicted viscosities for these mixtures obtained by using equation (3). The maximum difference between experimental viscosities and those predicted by equation (3) is 2.5 percent with an average difference of about 1 percent. Such results were taken as grounds for using the helium force constants in table 2 to calculate mixture viscosities of air and helium by the Hirschfelder method.

Heat capacity for binary mixtures.— The heat capacity of a binary mixture may be simply and accurately determined by

$$C_p = \sum_i C_{p_i} x_i = C_{p_1} x_1 + C_{p_2} x_2 \quad (4)$$

where  $C_{p_i}$  and  $x_i$  represent, respectively, the heat capacity and mole fraction of the  $i$ th component. The heat capacities for pure air and helium were taken from references 8 and 38.

#### Transport Properties Calculated for Air-Helium

##### Mixtures and Pure Gases

The coefficient of thermal conductivity for air-helium mixtures is given in table 4 and plotted in figure 7. For convenience, this has been expressed as a dimensionless conductivity following the practice of the National Bureau of Standards. The reference thermal conductivity is that of air at a temperature of 273.16 K and pressure of 1 atmosphere

and is taken as equal to  $5.760 \times 10^{-5}$  cal/(cm-sec)(°K).

Values of the conductivity are plotted at 100 degree increments from 200 to 3,000 K over the full mixture range at intervals of one-tenth mole fraction from pure air to pure helium. The results hold for dilute gases where the pressure effect is negligible.

The coefficient of viscosity for air-helium mixtures is given in table 5 and plotted in figure 8. Again as in the case of the thermal conductivity, the viscosity is reported as a ratio; the reference viscosity is that of air at a temperature of 273.16 K and pressure of 1 atmosphere and has the value  $1.715 \times 10^{-4}$  gm/sec-cm. The temperature and mole fraction arguments at which the viscosities are tabulated are the same as in the case of thermal conductivity.

The heat capacity at constant pressure for the mixtures of helium and air is given by table 6 as the dimensionless quantity  $C_p/R$ . Values are listed at the same increments of temperature and mole fractions as for the thermal conductivity.

The utility of the Prandtl number  $Pr = \eta C_p / k$  in heat-transfer considerations and the availability of heat-capacity, viscosity, and thermal-conductivity values reported in tables 4, 5, and 6 suggested that the Prandtl numbers at various temperatures and mole fractions can be calculated. The results are given in table 6 at the same temperatures and mole fractions used in the previous tables for thermal conductivity. The Prandtl

numbers are also plotted in figure 9, which covers the entire range of temperatures and mixture mole fractions. These data for the Prandtl number, as for the thermal conductivity, viscosity, and heat capacity, are applicable to dilute gases where pressure effects are negligible. To facilitate the use of the tables and charts of the dimensionless transport properties and heat capacities, conversion factors, table 3, have been included with both modes of data presentation.

Air Prandtl number.— Because of the importance of the Prandtl number of pure air in heat-transfer calculations, the results of this work were compared (fig. 10) with those of earlier studies (refs. 15, 39, 40, 41, and 43). The deviation of the Prandtl numbers is caused primarily by the different viscosity and conductivities used in the various works since the real-gas heat capacities are little different from the zero-pressure heat capacities used in some cases. Hansen (ref. 42) obtained predictions of the Prandtl number of air at high temperatures (to 15,000 K) by considering the molecules to behave as hard elastic spheres, the molecular collision integrals being represented as a simple function of the temperature according to the Sutherland formula. While this representation is consistent with other approximations employed at high temperatures, it is not so satisfactory as the collision-integral method. The latter method, which was used in the present study, yields quite accurate integrals for collisions between diatomic molecules.



The results reported in reference 42, which overlap those of this report, differ from those in this report by approximately 10 percent in the temperature range 1,000 to 3,000 K.

The uncertainty in the Prandtl number for air is less than 2 percent at temperatures less than 500 K and grows larger at higher temperatures where thermal-conductivity data are not so reliable. The results of this work agree with those of Hilsenrath and Touloukian (ref. 39) and Glassman and Bonilla (ref. 15) up to 500 K, but depart gradually (fig. 10) beyond this temperature until at 1,000 K it is less than the Hilsenrath-Touloukian values by 6.5 percent and less than the Glassman-Bonilla value by 3 percent. At this temperature this tabulation exceeds the Keenan-Kaye and Tribus-Boelter values by 1.4 percent. The tabulation of this work agrees to within 2.0 percent with the Prandtl number predictions made earlier (ref. 43) to 600 K. Beyond this temperature, the difference grows and reaches 5 percent at 1,500 K. A consideration of the properties used in the two studies to calculate the Prandtl number indicates that disagreement arises almost entirely from differences in the thermal conductivity predictions used in each case. The molecular parameters used in this work lead to larger thermal conductivities than those reported in reference 43 and cause a comparable reduction in the Prandtl number.

Helium Prandtl number.- Kinetic theory predicts a constant value for the Prandtl number of a monatomic gas. For the helium

molecular parameters used in this report, the Prandtl number was found to be 0.691. This compares with the value of 0.66 measured by Stroom et al. (ref. 45) and the range of values 0.67 to 0.73, determined by the correlations reported in reference 39 for the temperature range 100 to 600 K. The difference between Stroom's measurements and the 0.691 value calculated in this study is due to the somewhat smaller thermal conductivity values determined from the Lennard-Jones model for helium when compared with experimental data (ref. 45).

#### CONCLUDING REMARKS

Measurements of air Prandtl number have been obtained over the temperature range 270 to 675 K with a consistency of  $\pm 0.5$  percent. An error analysis of the experimental procedure and consideration of the effects of variable properties caused by temperature and composition variations indicate a maximum uncertainty of 2.5 percent for the data. These measurements, shown in figure 1, agree closely with the predictions made in reference 43. The predictions of this work are within 2.5 percent of the measured Prandtl number.

Various methods of predicting the thermal conductivity and viscosity of binary mixtures were investigated. These were combined with heat-capacity data to form Prandtl numbers. The method described by Hirschfelder, Curtiss, and Bird (ref. 7) using the Lennard-Jones intermolecular potential was selected using separately evaluated force constants from experimental thermal conductivity and viscosity data in calcu-

lating these properties. Mixture Prandtl numbers calculated by this method were within 3.0 percent of the helium-air experimental values reported in reference 43. Dimensionless thermal conductivity, viscosity, and Prandtl number are given in tables and charts over a temperature range of 200 to 3,000 K for mixtures ranging from pure air to pure helium.

University of Minnesota  
Minneapolis, Minnesota  
December 3, 1963

## REFERENCES

1. Eckert, E. R. G., and Irvine, T. F., Jr.: New Method to Measure Prandtl Number and Thermal Conductivity of Fluids. Jour. Appl. Mech., vol. 24, no. 1, Mar. 1957, pp. 25-28.
2. Pohlhausen, E.: Der Wärmeaustausch zwischen festen Körpern und Flüssigkeiten mit kleiner Reibung und kleiner Wärmeleitung. Z.a.M.M., Bd. 1, Heft 2, Apr. 1921, pp. 115-121.
3. Hayday, A., Irvine, T. F., Jr., and Eckert, E. R. G.: Thermal Diffusion Effects in Laminar Boundary Layers of Binary Mixtures. Heat Transfer Laboratory, Mechanical Engineering Department, University of Minnesota, April 21, 1959.
4. Lindsay, A. L., and Bromley, L. A.: Thermal Conductivity of Gas Mixtures. Ind. and Eng. Chem., vol. 42, no. 8, Aug. 1950, pp. 1508-1511.
5. Wassiljewa, A.: Wärmeleitung in Gasmischen. Physik Zeit., vol. 5, no. 22, Nov. 1904, pp. 737-742.
6. Keyes, F. G.: Measurements of the Heat Conductivity of Nitrogen-Carbon Dioxide Mixtures. Trans. ASME, vol. 73, 1951, pp. 597-603.
7. Hirschfelder, J. O., Curtiss, C. F., and Bird, R. B.: Molecular Theory of Gases and Liquids. John Wiley and Sons, Inc., 1954.
8. Hilsenrath, Joseph, Beckett, Charles W., et al.: Tables of Thermal Properties of Gases. Circular 564, Nat. Bur. Standards, Nov. 1, 1955.
9. Sherratt, G. G., and Griffiths, Ezer: A Hot Wire Method for the Thermal Conductivities of Gases. Phil. Mag. and Jour. Sci., ser. 7, vol. 27, no. 180, Jan. 1939, pp. 68-75.
10. Taylor, W. J., and Johnston, H. L.: An Improved Hot-Wire Cell for Accurate Measurements of Thermal Conductivities of Gases Over a Wide Temperature Range. Results With Air Between 87° and 375° K. Jour. Chem. Phys., vol. 14, no. 4, Apr. 1946, pp. 219-233.

11. Lambert, J. D., Staines, E. N., and Woods, S. D.: Thermal Conductivities of Organic Vapors. Proc. Roy. Soc. (London), ser. A, vol. 200, no. A 1061, Jan. 1950, pp. 262-271.
12. Stops, D. W.: The Effect of Temperature on the Thermal Conductivity of Gases. Nature, vol. 164, no. 4179, Dec. 3, 1949, pp. 966-967.
13. Boelter, L. M. K., and Sharp, W. H.: An Investigation of Aircraft Heaters. XXXII - Measurement of Thermal Conductivity of Air and of Exhaust Gases Between 50° and 900° F. NACA TN 1912, 1949.
14. Kannuluik, W. G., and Donald, H. B.: Aust. Jour. Sci. Res., ser. A, vol. 3, 1950, pp. 417-427.
15. Glassman, I., and Bonilla, C. F.: The Thermal Conductivity and Prandtl Number of Air at High Temperature. Chem. Eng. Prog., Symposium ser., vol. 49, 1953, pp. 153-162.
16. Irvine, T. F., Jr.: A New Method for the Experimental Determination of Prandtl Numbers and Thermal Conductivities of Gases. Results for Air. Ph.D. Thesis, Univ. of Minnesota, 1956.
17. Johnston, Herrick L., and Grilly, Edward R.: The Thermal Conductivities of Eight Common Gases Between 80° and 380° K. Jour. Chem. Phys., vol. 14, no. 4, Apr. 1946, pp. 233-238.
18. Kannuluik, W. G., and Carman, E. H.: The Thermal Conductivity of Rare Gases. Proc. Roy. Soc. (London), ser. B, vol. 65, pt. 9, Sept. 1952, pp. 701-709.
19. Thomas, L. B., and Golike, R. C.: A Comparative Study of Accommodation Coefficients by the Temperature-Jump and Low-Pressure Methods and Thermal Conductivities. Jour. Chem. Phys., vol. 22, no. 2, 1954, pp. 300-305.
20. Dickens, B. G.: The Effect of Accommodation Coefficients on Heat Conduction Through Gases. Proc. Roy. Soc. (London), ser. A, vol. 143, no. 850, Feb. 1934, pp. 517-540.
21. Buddenberg, J. W., and Wilke, C. R.: Calculation of Gas Mixture Viscosities. Ind. Eng. Chem., vol. 41, no. 7, July 1949, pp. 1345-1347.
22. Wilke, C. R.: A Viscosity Equation for Gas Mixtures. Jour. Chem. Phys., vol. 18, no. 4, Apr. 1950, pp. 517-519.

23. Carlson, C. O., and Schneider, F. J.: Transport Properties of Binary Gas Mixtures. Tech. Rep. No. 7, Contract AF-18(600)-1226, WADC and Univ. of Minnesota, Jan. 1956.
24. Bearden, J. A.: A Precision Determination of the Viscosity of Air. Phys. Rev., vol. 56, No. 1939, pp. 1023-1040.
25. Johnston, Herrick L., and McCloskey, Kenneth: Viscosities of Several Common Gases Between 90°K and Room Temperature. Jour. Phys. Chem., vol. 44, no. 9, Dec. 1940, pp. 1038-1058.
26. Wobser, R., and Müller, F.: Die innere Reibung von Gasen und Dämpfen und ihre Messung in Hoppler-Viskosimeter. Kolloid-Beihfte, Bd. LII, Heft 6-7, Feb. 4, 1941, pp. 165-276.
27. Vasilisco, Virgile: Recherches expérimentales sur la viscosité des gaz aux températures élevées. Ann. Phys., ser. 11, vol. 20, May-June 1945, pp. 137-176, 292-334.
28. Trautz, M., and Binkels, H. B.: Die Reibung, Wärmeleitung und Diffusion von Gasmischungen. Ann. Phys., Bd. 5, Heft 5, June 1930, pp. 561-580.
29. Trautz, M., and Zink, R.: Gasreibung bei höheren Temperaturen. Ann. Phys., Bd. 7, Heft 4, Dec. 3, 1930, pp. 427-452.
30. Trautz, M., and Hussein, I.: Die Reibung, Wärmeleitung und Diffusion von Gasmischungen. Ann. Phys., Bd. 20, Heft 2, June 1934, pp. 121-126.
31. Trautz, M., and Heberling, R.: Die Reibung, Wärmeleitung und Diffusion von Gasmischungen. Ann. Phys., Bd. 20, Heft 2, June 1934, pp. 118-121.
32. Trautz, M., and Zimmermann, H.: Die Reibung, Wärmeleitung und Diffusion von Gasmischungen. Ann. Phys., Bd. 22, Heft 2, Feb. 1935, pp. 189-194.
33. Johnston, H. L., and Grilly, E. R.: Viscosities of Carbon Monoxide, Helium, Neon and Argon Between 80° K and 300° K. Coefficients of Viscosity. Jour. Phys. Chem., vol. 46, no. 8, Nov. 1942, pp. 948-963.
34. Rietveld, A. O., Van Itterbeek, A., and Van Den Berg, G. J.: Measurements on the Viscosity of Mixtures of Helium and Argon. Physica, vol. 19, May 1953, pp. 517-524.

35. Kestin, J., and Pilarczyk, K.: Measurement of the Viscosity of Five Gases at Elevated Pressures by the Oscillating-Disk Method. Trans. ASME, vol. 76, no. 6, Aug. 1954, pp. 987-997; discussion, pp. 998-999.
36. Heath, H. R.: The Viscosity of Gas Mixtures. Proc. Phys. Soc. (London), ser. B, vol. 66, pt. 5, May 1953, pp. 362-367.
37. Trautz, M., and Kipphan, K. F.: Die Reibung, Wärmeleitung und Diffusion von Gasmischungen. Ann. Phys., Bd. 2, Heft 6, Aug. 1929, pp. 743-748.
38. Woolley, Harold W.: Thermodynamic Properties of Gaseous Nitrogen. NACA TN 3271, 1956.
39. Hilsenrath, Joseph, and Touloukian, Y. S.: The Viscosity Thermal Conductivity and Prandtl Number for Air,  $O_2$ ,  $N_2$ , NO,  $H_2$ , CO,  $CO_2$ ,  $H_2O$ , He, and A. Trans. ASME, vol. 76, 1954, pp. 967-983; discussion, 983-985.
40. Keenan, J. H., and Kaye, J.: Gas Tables. John Wiley and Sons, Inc., 1948.
41. Tribus, M., and Boelter, L. M. K.: An Investigation of Aircraft Heaters. II - Properties of Gas. NACA WR W-9, 1942.
42. Hansen, C. F.: Approximations for the Thermodynamic and Transport Properties of High-Temperature Air. NACA TN 4150, 1958.
43. Eckert, E. R. G., Ibele, W. E., and Irvine, T. F. Jr.: Prandtl Number, Thermal Conductivity and Viscosity of Air-Helium Mixtures. NASA TN D-533, 1960.
44. Novotny, J. L., Irvine, T. F. Jr.: Thermal Conductivity and Prandtl Number of Carbon Dioxide and Carbon-Dioxide Air Mixtures at One Atmosphere. Trans. ASME, vol. 83, series C, No. 2, 1961, pp. 125-132.
45. Stroom, P. D., Ibele, W. E., and Irvine, Thomas F. Jr.: Helium Prandtl Number Measurements and Calculated Viscosity and Thermal Conductivity: "International Developments in Heat Transfer" 1961-62 Am. Soc. Mech. Eng.
46. Dahl, A. I., The Stability of Base-metal Thermocouples in Air from 800 to 2200°F. Temperature Its Measurement and Control in Science and Industry, Am. Inst. of Phy. Reinhold 1941. pp. 1238-1266.

47. Lindahl, B. C.: Thermal Conductivity Predictions for Binary Gas Mixtures, Master of Science Thesis, University of Minnesota, 1959.
48. Hirschfelder, J. O.: Heat Transfer in Chemically Reacting Gas Mixture, J. Chem. Phys. 26, 1957, pp. 282-285.
49. Brokaw, R. S.: Estimating Thermal Conductivities for Nonpolar Gas Mixtures. Ind. Eng. Chem. 47, 1955, p. 2398.
50. Pulkrabek, W. W.: Prandtl Number Measurements for Helium Air Mixtures, Master of Science Thesis, University of Minnesota, September 1963.
51. Geier, H., and Schafer, K.: Thermal Conductivity of Pure Gases and Gas Mixtures Between 0° and 1200°C., Allgemeine Wärmetechnik, band 10, heft 4, 1961, pp. 70-75.
52. Timrot, D. L., and Vargaftik, N. B.: Heat Conductivity, Viscosity and Thermodynamical Properties of Steam at High Temperatures and Pressures, Fifth International Conference on Properties of Steam, London, 1955.
53. Vines, R. G.: Measurement of Thermal Conductivities of Gases at High Temperatures. Trans. ASME, vol. 82, Series C, No. 1, 1960, pp. 48-52.



TABLE 1.- VALUES OF PRANDTL NUMBER  
OF AIR FROM FAIRED CURVE

Temperature Degrees <u>Kelvin</u>	Prandtl <u>Number</u>
280	0.711
300	0.708
320	0.705
340	0.702
360	0.700
380	0.698
400	0.696
420	0.694
440	0.692
460	0.690
480	0.689
500	0.688
520	0.687
540	0.686
560	0.685
580	0.685
600	0.685
620	0.685
640	0.686
660	0.686
680	0.687

TABLE 2. INTERMOLECULAR FORCE CONSTANTS

Gas	From viscosity data (ref. 7)		From thermal conductivity data	
	$\sigma, \text{\AA}$	$\epsilon/k, ^\circ\text{K}$	$\sigma, \text{\AA}$	$\epsilon/k, ^\circ\text{K}$
Helium	2.576	10.22	2.585	12.5
Air	3.689	84.0	3.421	154.

TABLE 3. CONVERSION FACTORS FOR THERMAL CONDUCTIVITY,  
VISCOSITY, AND HEAT CAPACITY

To change -	To -	Having dimensions -	Multiply by -
Thermal conductivity			
$k/k_o$	$k$	Cal/(sec)(cm)(°K) Watts/(cm)(°K) Btu/(hr)(ft.)(°R)	$0.5760 \times 10^{-4}$ 2.410 139.3
Viscosity			
$\eta/\eta_o$	$\eta$	Kg/(hr)(m) Slug/(hr)(ft) <sup>2</sup> Lb <sub>f</sub> (sec)/(ft) <sup>2</sup> Gm/(sec)(cm) Lb <sub>m</sub> /(hr)(ft) Lb <sub>m</sub> (sec)(ft)	$6,174.0 \times 10^{-5}$ 128.9 0.03582 17.15 4,149.0 1.152
Heat capacity			
$C_p/R$	$C_p$	Cal/(gm-mole)(°K) Btu/(lb-mole)(°R)	1.98719 1.98588

TABLE 4. THERMAL CONDUCTIVITY OF AIR-HELIUM MIXTURES AT 1-ATMOSPHERE PRESSURE

T, °K	k/k <sub>0</sub> for mole fraction of helium of =										T, °R	
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
200	.7513	.9442	1.156	1.392	1.657	1.960	2.313	2.733	3.245	3.890	4.738	360
300	1.085	1.329	1.598	1.897	2.235	2.623	3.074	3.612	4.268	5.094	6.176	540
400	1.385	1.673	1.992	2.48	2.750	3.211	3.749	4.390	5.172	6.155	7.442	720
500	1.668	1.998	2.363	2.771	3.232	3.761	4.377	5.111	6.005	7.129	8.598	900
600	1.943	2.314	2.724	3.182	3.698	4.290	4.978	5.797	6.792	8.041	9.670	1,080
700	2.213	2.627	3.082	3.589	4.160	4.812	5.570	6.466	7.553	8.913	10.68	1,260
800	2.477	2.933	3.434	3.991	4.615	5.326	6.149	7.119	8.292	9.752	11.64	1,440
900	2.735	3.233	3.779	4.383	5.058	5.825	6.710	7.750	9.003	10.56	12.56	1,620
1,000	2.988	3.525	4.112	4.760	5.484	6.304	7.247	8.354	9.683	11.33	13.44	1,800
1,100	3.235	3.810	4.438	5.130	5.901	6.773	7.773	8.943	10.35	12.08	14.30	1,980
1,200	3.477	4.090	4.759	5.494	6.312	7.233	8.289	9.520	10.99	12.80	15.12	2,160
1,300	3.717	4.369	5.077	5.855	6.718	7.688	8.795	10.08	11.62	13.51	15.91	2,340
1,400	3.954	4.642	5.390	6.208	7.113	8.129	9.286	10.63	12.23	14.18	16.66	2,520
1,500	4.191	4.916	5.701	6.559	7.507	8.568	9.774	11.17	12.82	14.84	17.40	2,700
1,600	4.426	5.189	6.011	6.908	7.894	8.993	10.24	11.67	13.35	15.40	17.97	2,880
1,700	4.663	5.463	6.324	7.259	8.282	9.418	10.70	12.16	13.88	15.94	18.53	3,060
1,800	4.905	5.743	6.642	7.614	8.674	9.845	11.16	12.65	14.39	16.48	19.06	3,240
1,900	5.161	6.037	6.974	7.983	9.079	10.28	11.63	13.15	14.91	17.01	19.58	3,420
2,000	5.438	6.355	7.331	8.377	9.510	10.75	12.12	13.67	15.44	17.54	20.09	3,600
2,100	5.746	6.707	7.724	8.809	9.977	11.25	12.64	14.21	15.99	18.07	20.59	3,780
2,200	6.116	7.124	8.187	9.314	10.52	11.82	13.24	14.81	16.58	18.63	21.07	3,960
2,300	6.573	7.639	8.753	9.927	11.17	12.50	13.93	15.50	17.24	19.23	21.55	4,140
2,400	7.134	8.266	9.441	10.67	11.95	13.30	14.74	16.29	17.98	19.86	22.01	4,320
2,500	7.831	9.043	10.29	11.57	12.90	14.27	15.71	17.22	18.83	20.56	22.47	4,500
2,600	8.708	10.02	11.35	12.69	14.07	15.46	16.89	18.34	19.82	21.34	22.91	4,680
2,700	9.805	11.23	12.66	14.09	15.51	16.92	18.31	19.68	20.99	22.23	23.35	4,860
2,800	11.15	12.71	14.26	15.78	17.26	18.68	20.03	21.26	22.35	23.22	23.78	5,040
2,900	12.79	14.53	16.21	17.84	19.38	20.80	22.08	23.16	23.95	24.36	24.20	5,220
3,000	14.65	16.58	18.42	20.16	21.76	23.19	24.39	25.27	25.73	25.59	24.61	5,400

TABLE 5. VISCOSITY OF AIR-HELIUM MIXTURES AT 1-ATMOSPHERE PRESSURE

T, °K	$\eta/\eta_0$ for mole fraction helium of -										T, °R	
	0)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
200	.7845	.7991	.8147	.8311	.8481	.8654	.8822	.8972	.9077	.9086	.8897	360
300	1.072	1.087	1.104	1.121	1.138	1.156	1.172	1.185	1.192	1.186	1.156	540
400	1.316	1.332	1.350	1.368	1.386	1.404	1.420	1.432	1.438	1.429	1.393	720
500	1.534	1.552	1.572	1.592	1.612	1.631	1.648	1.661	1.665	1.652	1.608	900
600	1.734	1.754	1.776	1.798	1.820	1.841	1.860	1.873	1.877	1.861	1.809	1,080
700	1.920	1.943	1.966	1.990	2.014	2.036	2.056	2.070	2.073	2.055	1.998	1,260
800	2.096	2.121	2.146	2.172	2.198	2.222	2.243	2.258	2.261	2.241	2.178	1,440
900	2.260	2.287	2.314	2.342	2.371	2.397	2.421	2.437	2.440	2.418	2.349	1,620
1,000	2.416	2.445	2.474	2.505	2.535	2.564	2.590	2.608	2.611	2.588	2.515	1,800
1,100	2.568	2.599	2.631	2.663	2.695	2.726	2.753	2.772	2.775	2.750	2.670	1,980
1,200	2.717	2.750	2.783	2.818	2.852	2.884	2.912	2.931	2.934	2.906	2.820	2,160
1,300	2.864	2.898	2.933	2.969	3.004	3.038	3.067	3.087	3.089	3.058	2.967	2,340
1,400	3.007	3.043	3.079	3.117	3.154	3.189	3.219	3.239	3.241	3.208	3.111	2,520
1,500	3.148	3.185	3.223	3.262	3.300	3.336	3.367	3.388	3.389	3.355	3.253	2,700
1,600	3.286	3.324	3.363	3.403	3.441	3.477	3.507	3.524	3.520	3.477	3.360	2,880
1,700	3.421	3.460	3.500	3.540	3.579	3.614	3.643	3.658	3.648	3.595	3.463	3,060
1,800	3.547	3.588	3.629	3.671	3.710	3.746	3.774	3.786	3.772	3.711	3.564	3,240
1,900	3.672	3.715	3.757	3.800	3.840	3.876	3.902	3.913	3.894	3.824	3.661	3,420
2,000	3.796	3.839	3.883	3.927	3.968	4.003	4.029	4.036	4.013	3.934	3.756	3,600
2,100	3.917	3.962	4.007	4.052	4.093	4.128	4.153	4.158	4.129	4.041	3.849	3,780
2,200	4.038	4.084	4.130	4.175	4.217	4.252	4.275	4.277	4.243	4.146	3.940	3,960
2,300	4.156	4.203	4.251	4.296	4.338	4.373	4.395	4.394	4.355	4.249	4.028	4,140
2,400	4.273	4.322	4.370	4.416	4.458	4.493	4.513	4.509	4.465	4.351	4.115	4,320
2,500	4.389	4.438	4.487	4.534	4.577	4.611	4.630	4.623	4.573	4.450	4.200	4,500
2,600	4.501	4.551	4.602	4.649	4.692	4.726	4.744	4.734	4.679	4.548	4.283	4,680
2,700	4.611	4.663	4.714	4.763	4.806	4.839	4.856	4.844	4.784	4.643	4.364	4,860
2,800	4.720	4.773	4.823	4.875	4.919	4.951	4.966	4.951	4.887	4.738	4.444	5,040
2,900	4.828	4.883	4.936	4.986	5.030	5.062	5.076	5.058	4.988	4.831	4.523	5,220
3,000	4.935	4.991	5.045	5.095	5.139	5.171	5.183	5.162	5.087	4.921	4.600	5,400

TABLE 6. HEAT CAPACITY OF AIR-HELIUM MIXTURES AT 1-ATMOSPHERE PRESSURE

T, °K	$C_p/R$ for mole fraction helium of -										T, °R
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
200	3.506	3.406	3.305	3.204	3.104	3.003	2.902	2.802	2.701	2.601	2.500
300	3.506	3.405	3.305	3.204	3.104	3.003	2.902	2.802	2.701	2.601	540
400	3.533	3.430	3.327	3.223	3.120	3.017	2.913	2.810	2.707	2.603	720
500	3.588	3.479	3.371	3.262	3.153	3.044	2.935	2.826	2.718	2.609	900
600	3.663	3.546	3.430	3.314	3.198	3.081	2.965	2.849	2.733	2.616	1,080
700	3.746	3.621	3.496	3.372	3.247	3.123	2.998	2.874	2.749	2.625	1,260
800	3.828	3.695	3.562	3.430	3.297	3.164	3.031	2.898	2.766	2.633	1,440
900	3.906	3.765	3.625	3.484	3.344	3.203	3.062	2.922	2.781	2.641	1,620
1,000	3.979	3.831	3.683	3.535	3.387	3.240	3.092	2.944	2.796	2.648	1,800
1,100	4.046	3.891	3.737	3.582	3.428	3.273	3.118	2.964	2.809	2.655	1,980
1,200	4.109	3.948	3.787	3.626	3.465	3.305	3.144	2.983	2.822	2.661	2,160
1,300	4.171	4.004	3.837	3.670	3.503	3.336	3.168	3.001	2.834	2.667	2,340
1,400	4.230	4.057	3.884	3.711	3.538	3.365	3.192	3.019	2.846	2.673	2,520
1,500	4.289	4.110	3.931	3.752	3.573	3.395	3.216	3.037	2.858	2.679	2,700
1,600	4.352	4.167	3.982	3.796	3.611	3.426	3.241	3.056	2.870	2.685	2,880
1,700	4.418	4.226	4.034	3.843	3.651	3.459	3.267	3.075	2.884	2.692	3,060
1,800	4.487	4.288	4.090	3.891	3.692	3.494	3.295	3.096	2.897	2.699	3,240
1,900	4.566	4.359	4.153	3.946	3.740	3.533	3.326	3.120	2.913	2.707	3,420
2,000	4.662	4.446	4.230	4.013	3.797	3.581	3.365	3.149	2.932	2.716	3,600
2,100	4.781	4.553	4.325	4.097	3.869	3.641	3.412	3.184	2.956	2.728	3,780
2,200	4.947	4.702	4.458	4.213	3.968	3.724	3.479	3.234	2.989	2.745	3,960
2,300	5.179	4.911	4.643	4.375	4.107	3.840	3.572	3.304	3.036	2.768	4,140
2,400	5.484	5.186	4.887	4.589	4.290	3.992	3.694	3.395	3.097	2.798	4,320
2,500	5.882	5.544	5.206	4.867	4.529	4.191	3.853	3.515	3.176	2.838	4,500
2,600	6.40	6.010	5.620	5.230	4.840	4.450	4.060	3.670	3.280	2.890	4,680
2,700	7.06	6.604	6.148	5.692	5.236	4.780	4.324	3.868	3.412	2.956	4,860
2,800	7.87	7.333	6.796	6.259	5.722	5.185	4.648	4.111	3.574	3.037	5,040
2,900	8.86	8.224	7.588	6.952	6.316	5.680	5.044	4.408	3.772	3.136	5,220
3,000	9.96	9.214	8.468	7.722	6.976	6.230	5.484	4.738	3.992	3.246	5,400

TABLE 7. PRANDTL NUMBER OF AIR-HELIUM MIXTURES AT 1-ATMOSPHERE PRESSURE

T, °K	$\eta C_p/k$ for mole fraction helium of -										T, °R	
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
200	0.748	.644	.575	.527	.495	.476	.468	.474	.497	.553	.691	360
300	.707	.623	.563	.522	.493	.475	.468	.473	.496	.551	.691	540
400	.686	.610	.556	.517	.490	.478	.468	.472	.496	.550	.691	720
500	.674	.604	.553	.516	.490	.474	.468	.473	.496	.550	.691	900
600	.667	.601	.552	.516	.490	.474	.468	.474	.496	.551	.691	1,080
700	.664	.599	.550	.515	.490	.474	.468	.474	.496	.551	.691	1,260
800	.662	.597	.549	.514	.489	.474	.468	.473	.496	.551	.691	1,440
900	.659	.595	.548	.513	.488	.473	.467	.473	.496	.551	.691	1,620
1,000	.657	.594	.547	.512	.488	.473	.467	.473	.496	.551	.691	1,800
1,100	.656	.593	.547	.512	.488	.473	.467	.473	.496	.550	.691	1,980
1,200	.656	.593	.547	.512	.488	.473	.467	.473	.495	.550	.681	2,160
1,300	.656	.594	.547	.513	.488	.473	.467	.473	.496	.550	.681	2,340
1,400	.657	.594	.548	.513	.489	.474	.468	.474	.496	.551	.691	2,520
1,500	.658	.595	.548	.514	.490	.474	.469	.474	.497	.551	.691	2,700
1,600	.660	.597	.550	.515	.491	.475	.470	.475	.498	.552	.691	2,880
1,700	.662	.598	.551	.516	.492	.476	.471	.476	.499	.553	.691	3,060
1,800	.663	.599	.551	.517	.492	.477	.471	.477	.499	.553	.691	3,240
1,900	.664	.599	.552	.517	.493	.478	.472	.478	.500	.554	.691	3,420
2,000	.665	.600	.553	.518	.494	.479	.473	.479	.501	.555	.691	3,600
2,100	.666	.601	.554	.519	.495	.480	.474	.480	.502	.555	.691	3,780
2,200	.667	.602	.555	.520	.496	.481	.475	.481	.503	.556	.691	3,960
2,300	.669	.604	.556	.522	.497	.482	.477	.482	.504	.557	.691	4,140
2,400	.671	.606	.558	.523	.499	.484	.478	.484	.506	.558	.691	4,320
2,500	.673	.608	.560	.525	.501	.486	.480	.486	.507	.559	.691	4,500
2,600	.676	.610	.563	.528	.503	.488	.482	.488	.509	.561	.691	4,680
2,700	.678	.613	.565	.530	.506	.491	.485	.490	.511	.562	.691	4,860
2,800	.681	.615	.567	.533	.508	.493	.488	.493	.514	.564	.691	5,040
2,900	.683	.618	.570	.535	.511	.496	.490	.496	.517	.566	.691	5,220
3,000	.685	.620	.572	.538	.513	.499	.493	.498	.519	.568	.691	5,400

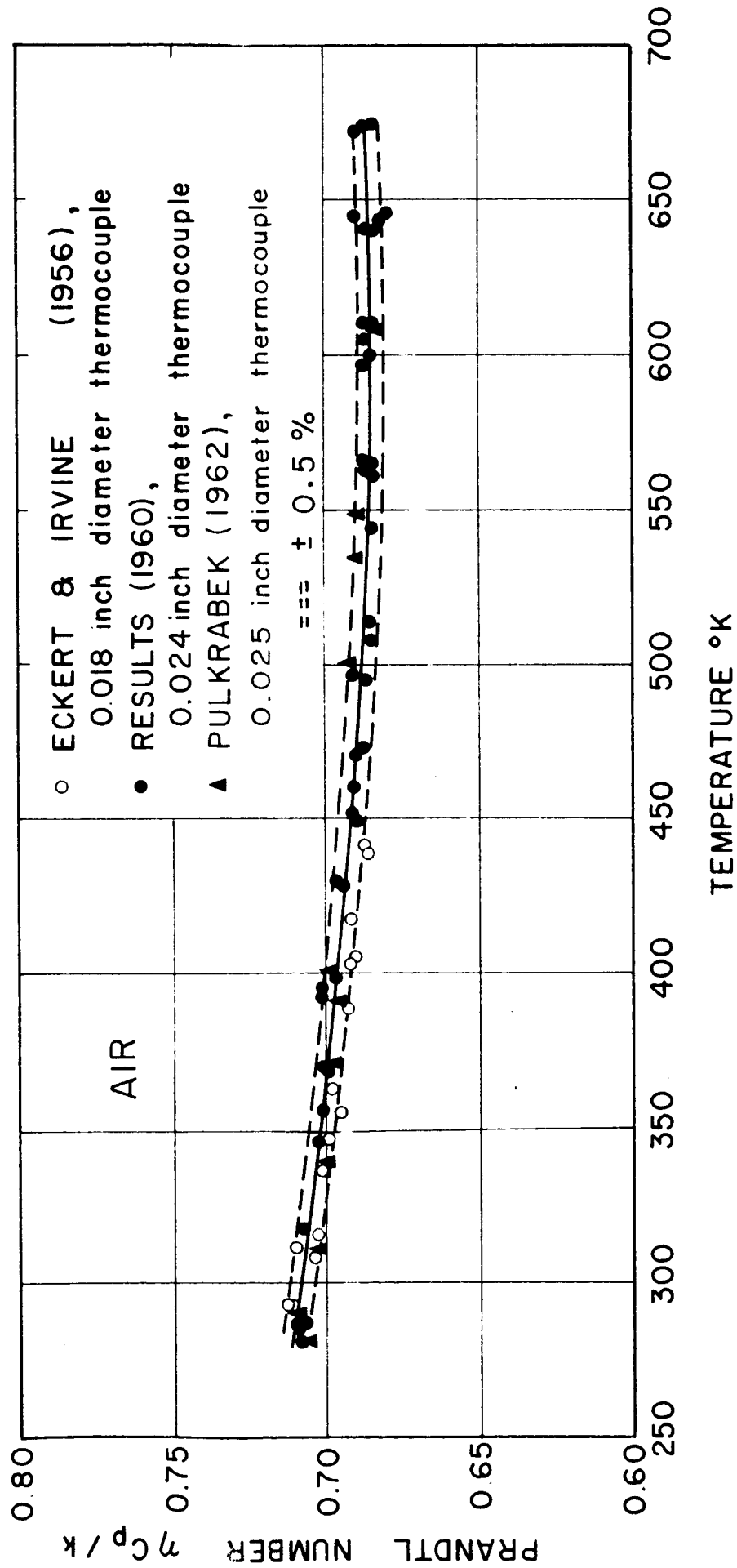


FIGURE 1



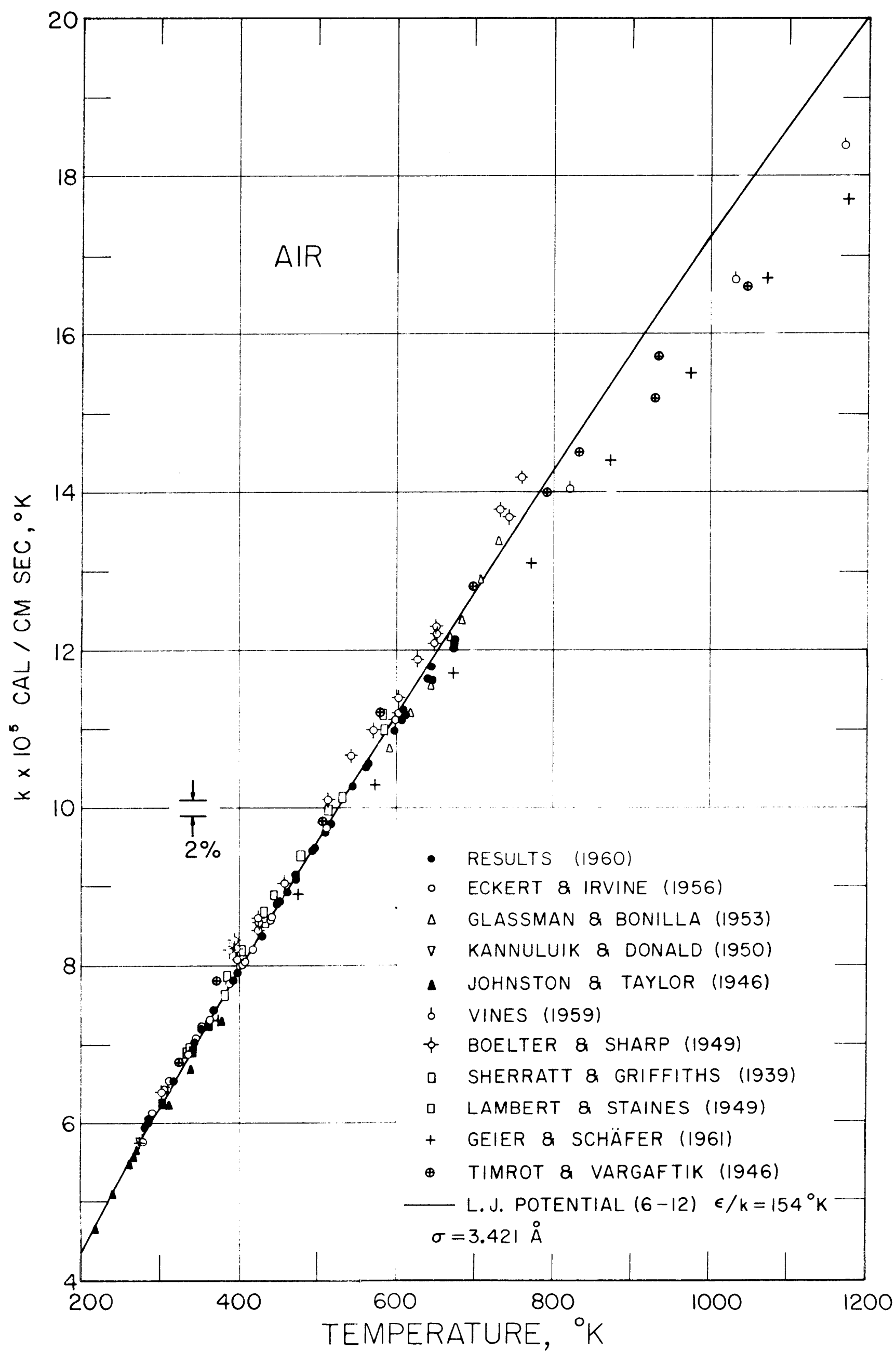
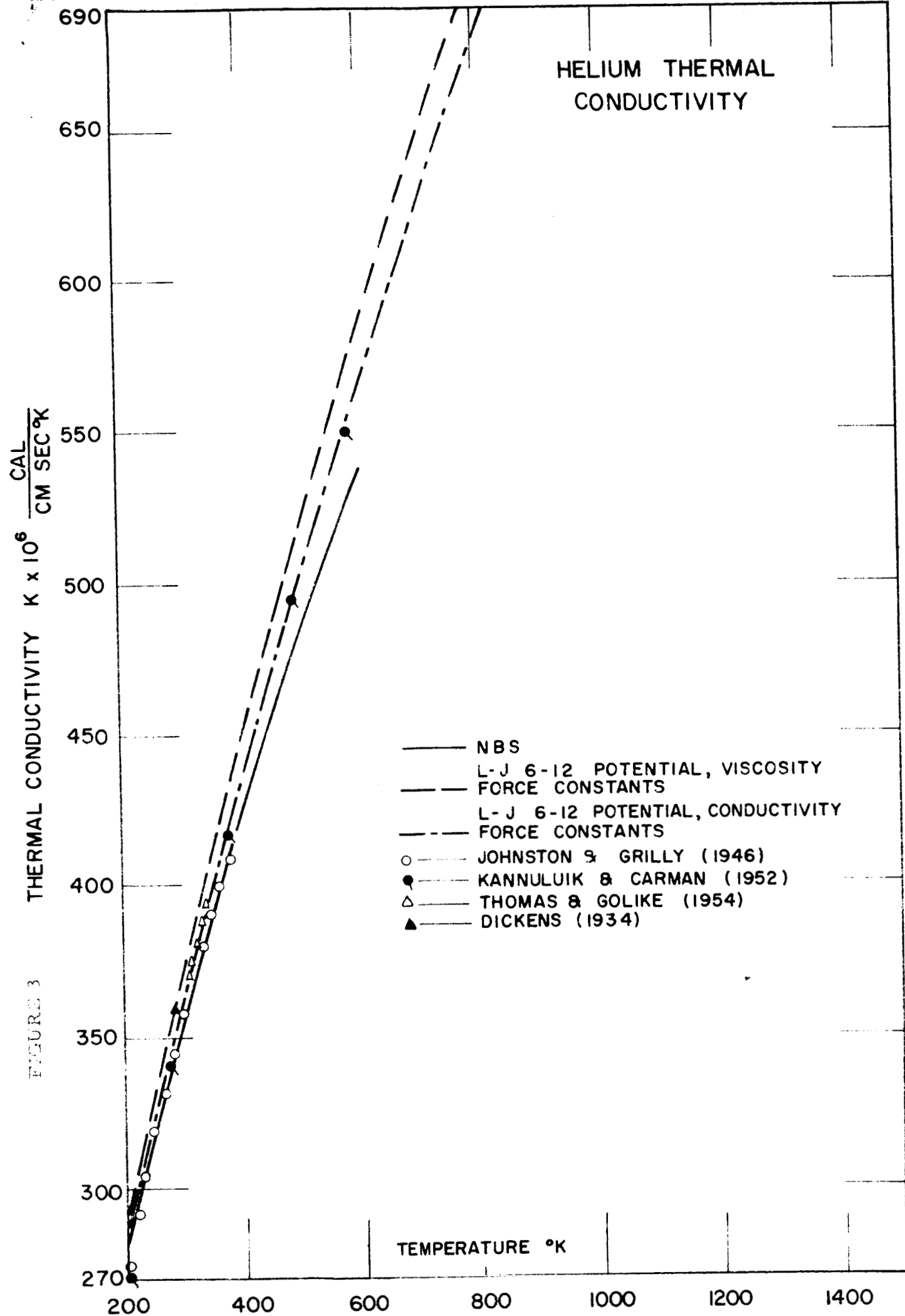


FIGURE 2



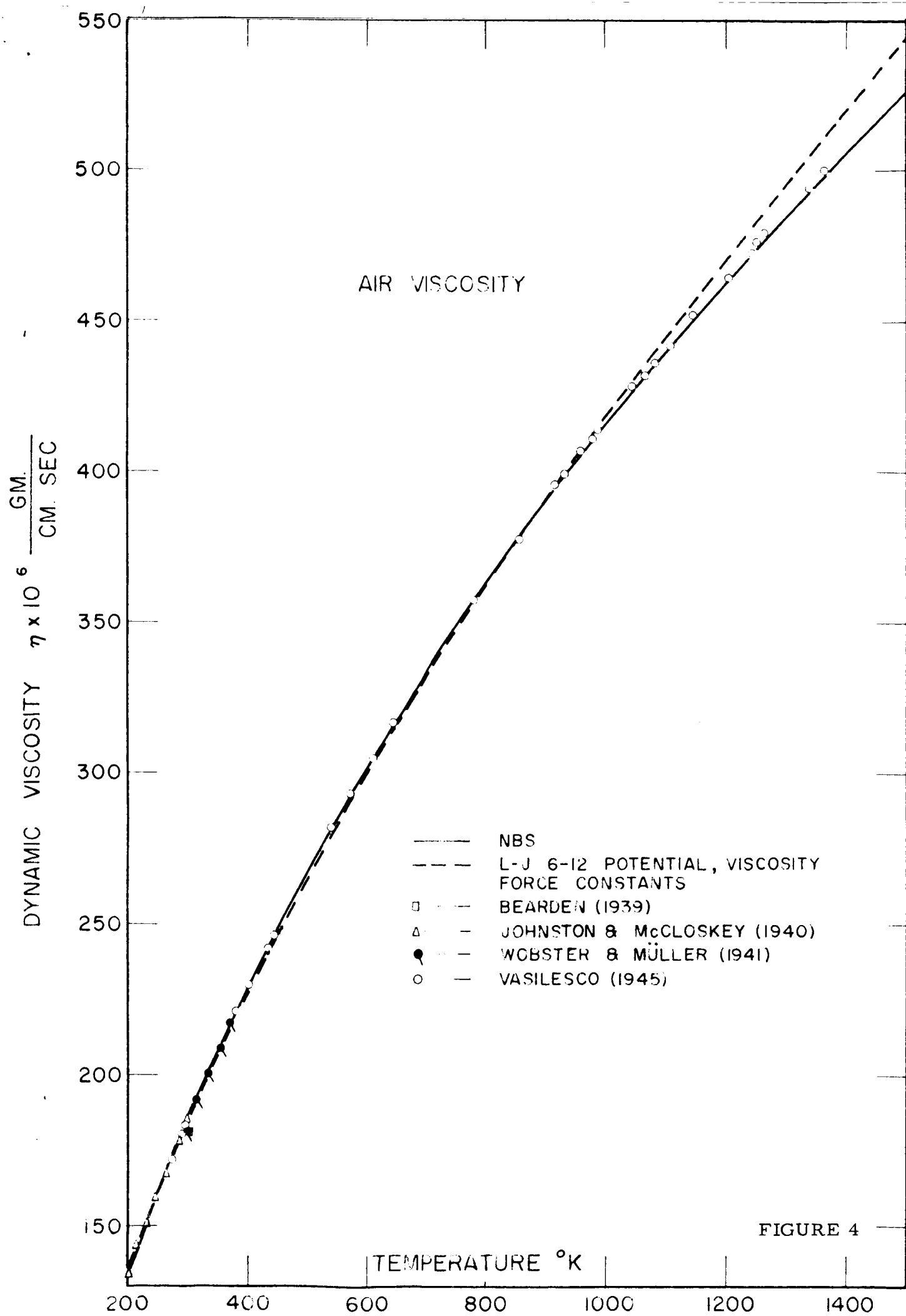


FIGURE 4

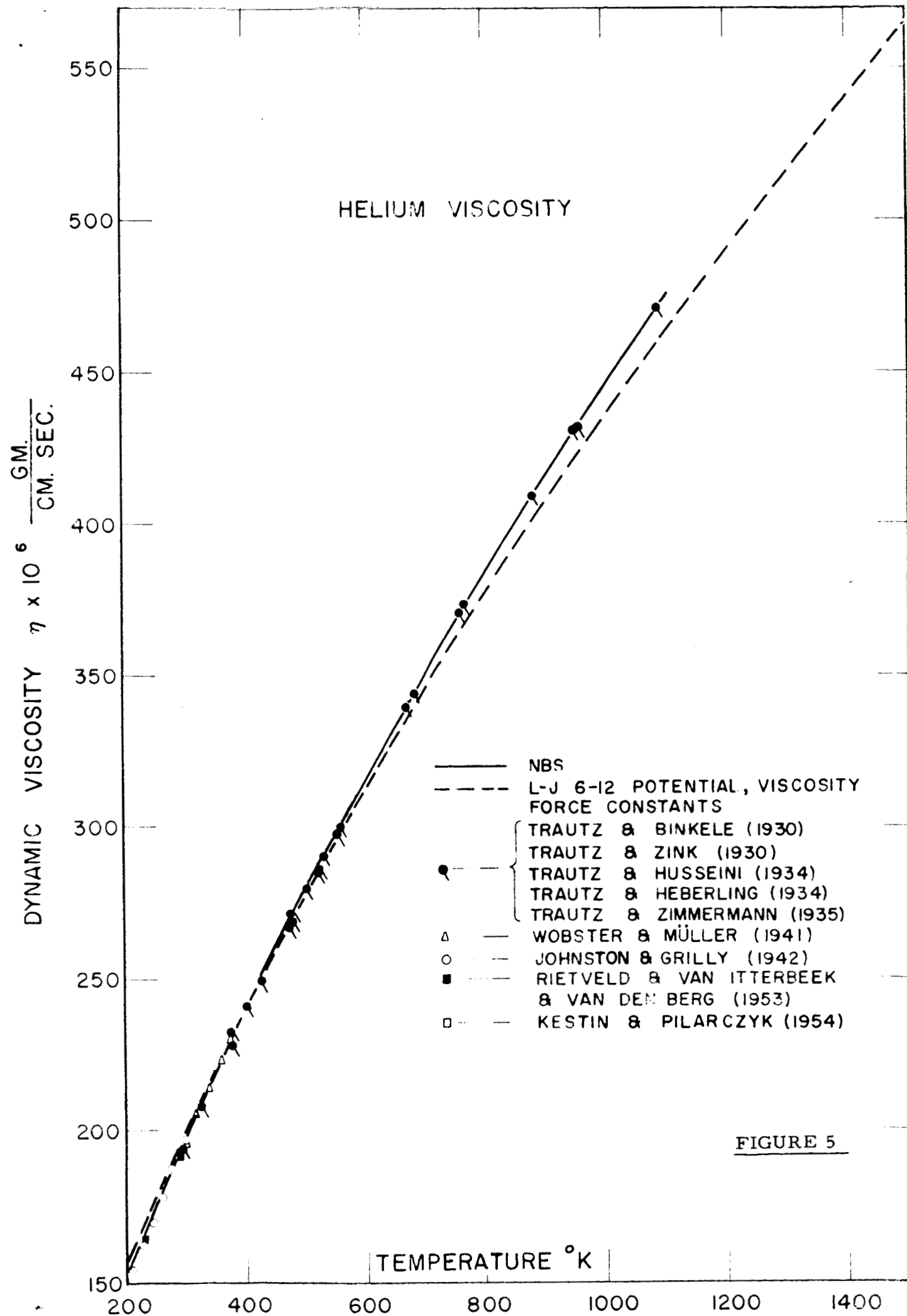


FIGURE 5

# BINARY GAS MIXTURE VISCOSITIES

- CALCULATED LJ 6-12 POTENTIAL  
 EXPERIMENTAL DATA  
 □ TRAUTZ, KIPPHAN (1929)  
 • TRAUTZ, BINKELE (1930)  
 \ TRAUTZ, BINKELE (1930)  
 ° HEATH (1953)  
 Δ HEATH (1953)  
 ρ RIETVELD, VAN ITTERBEEK, VAN DEN BERG (1953)

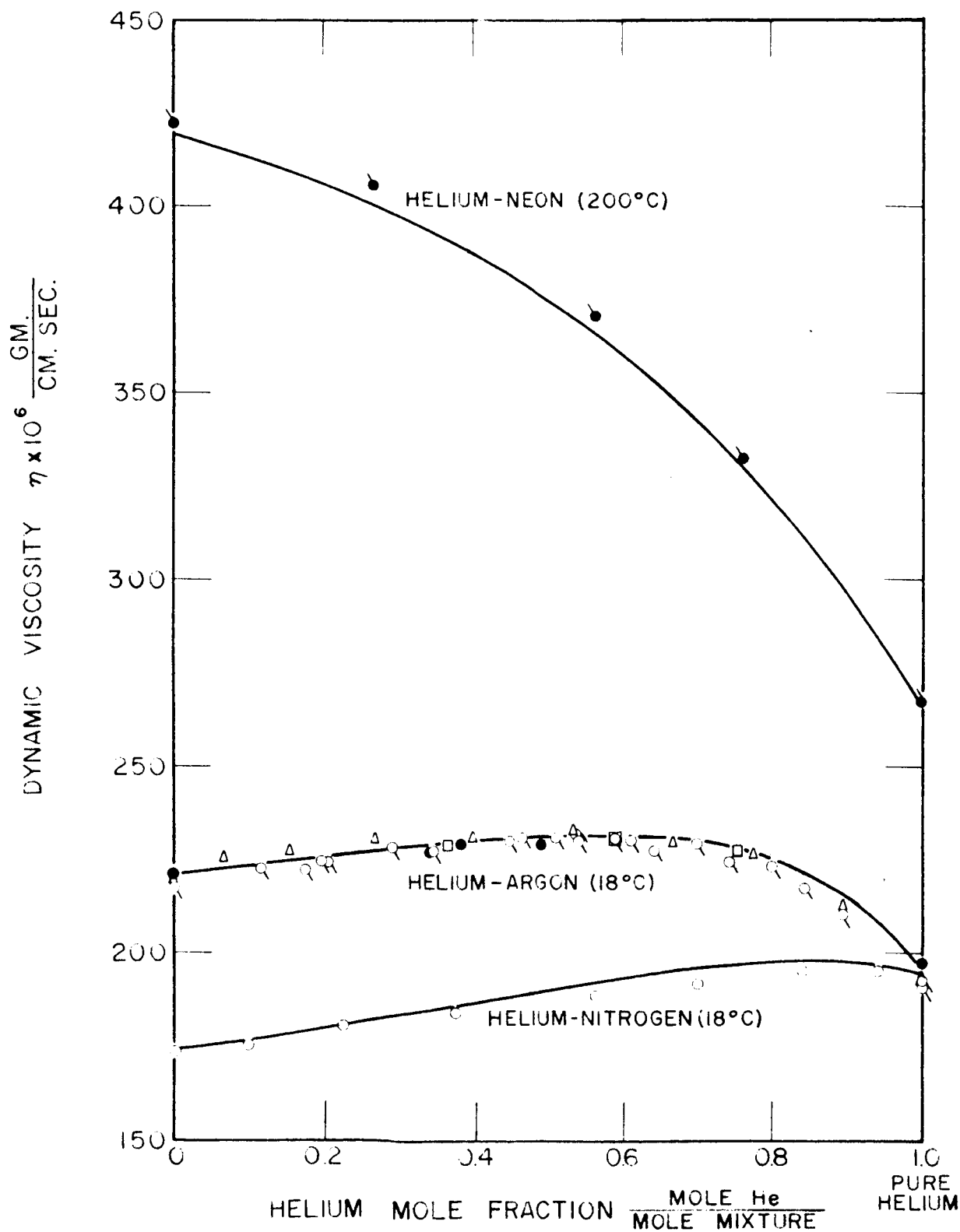
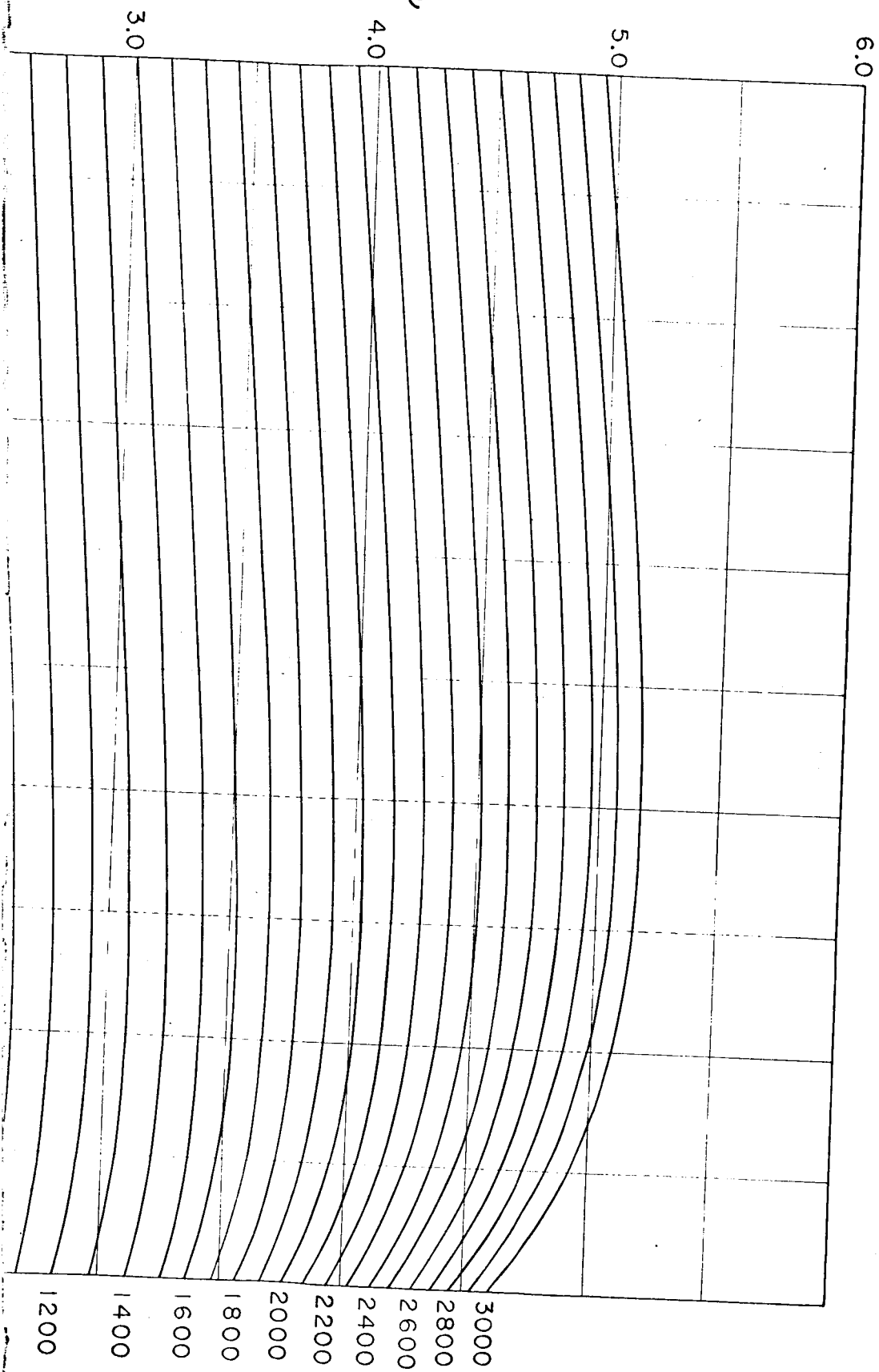


FIGURE 6

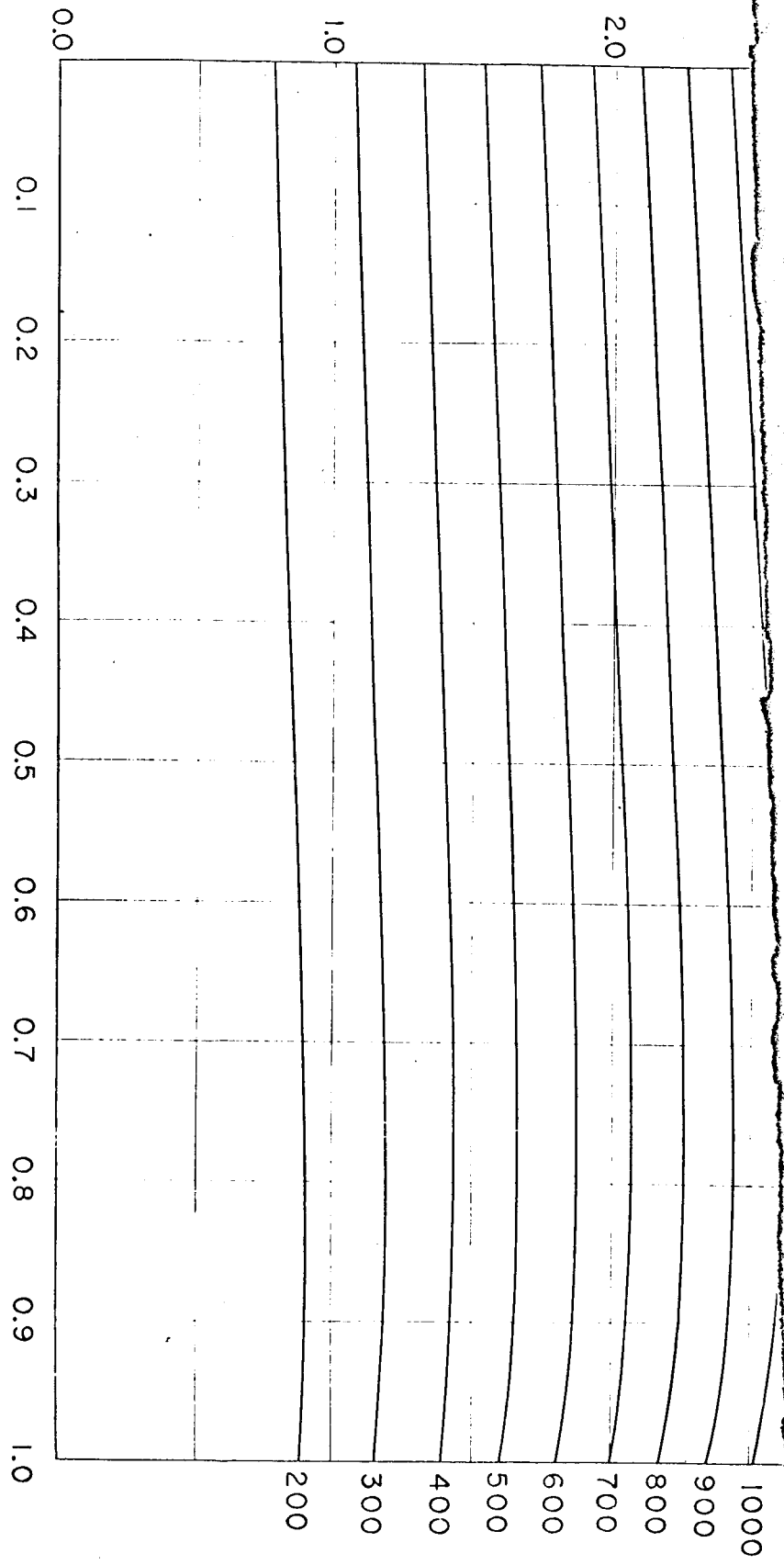
13

SITY RATIO -  $\eta/\eta_0$



°K

VISCO



HELIUM MOLE FRACTION - ( $\frac{\text{MOLE HE}}{\text{MOLE MIXTURE}}$ )

17

2A

L NUMBER

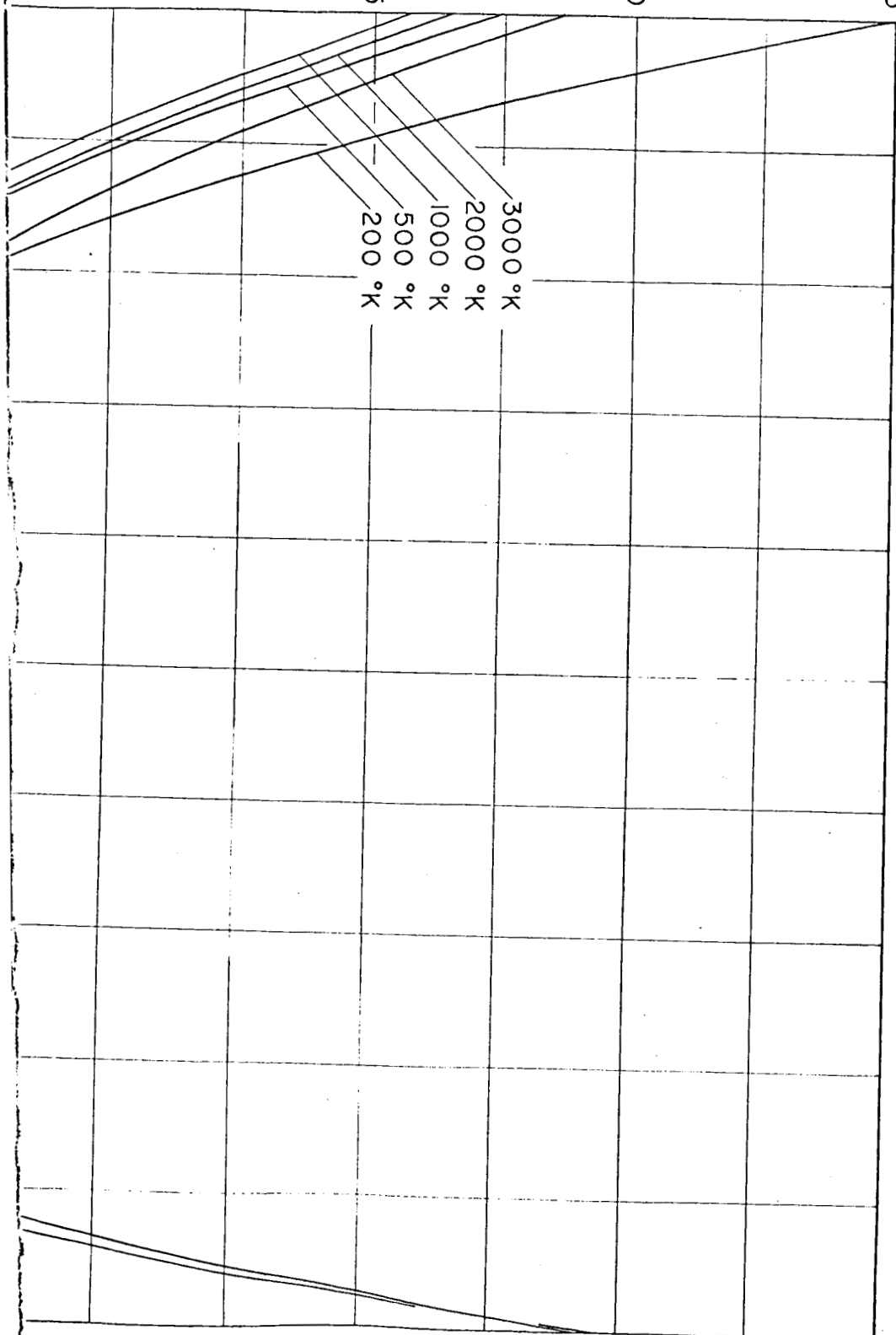
0.75

0.70

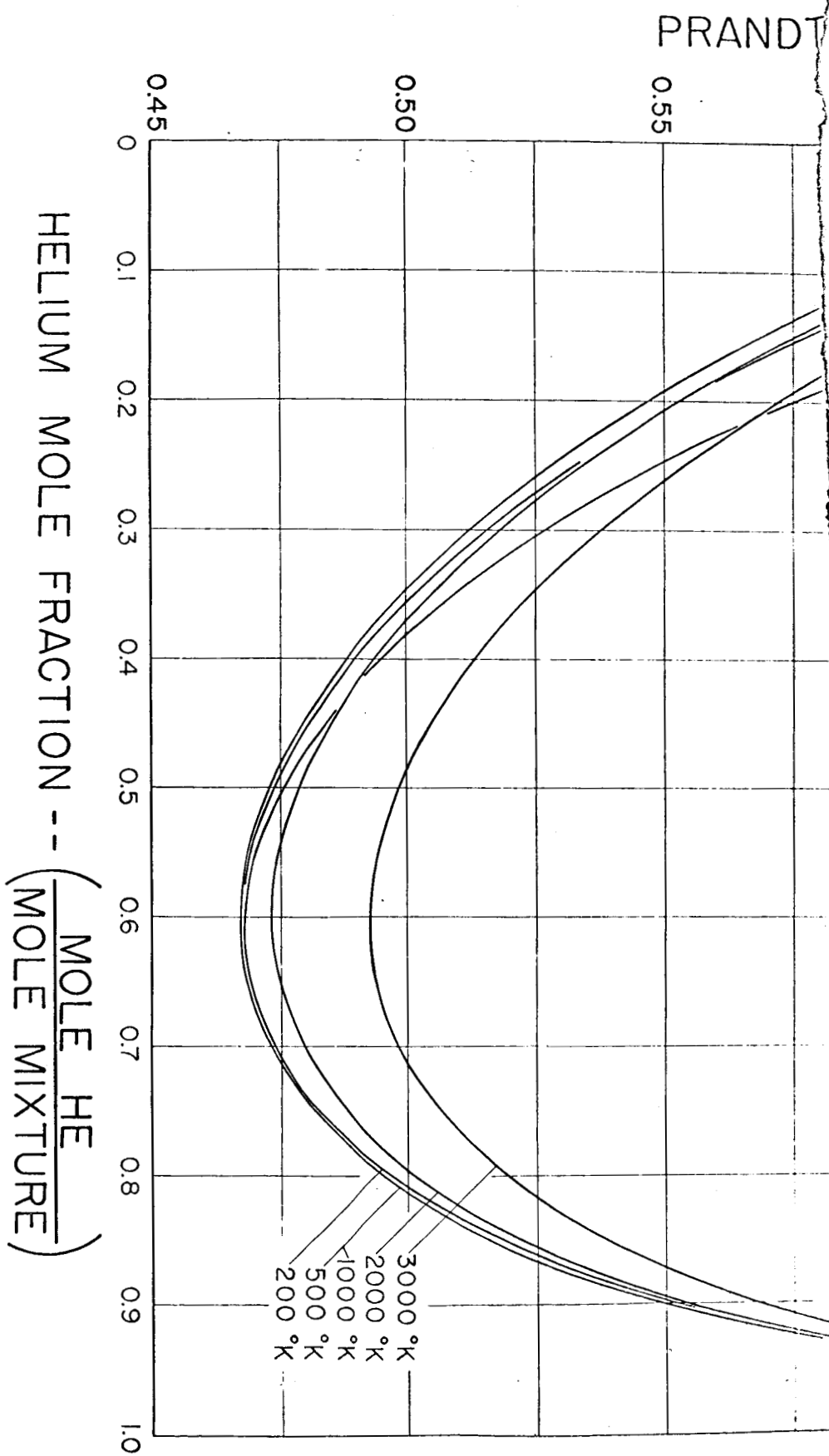
0.65

0.60

3000 °K  
2000 °K  
1000 °K  
500 °K  
200 °K



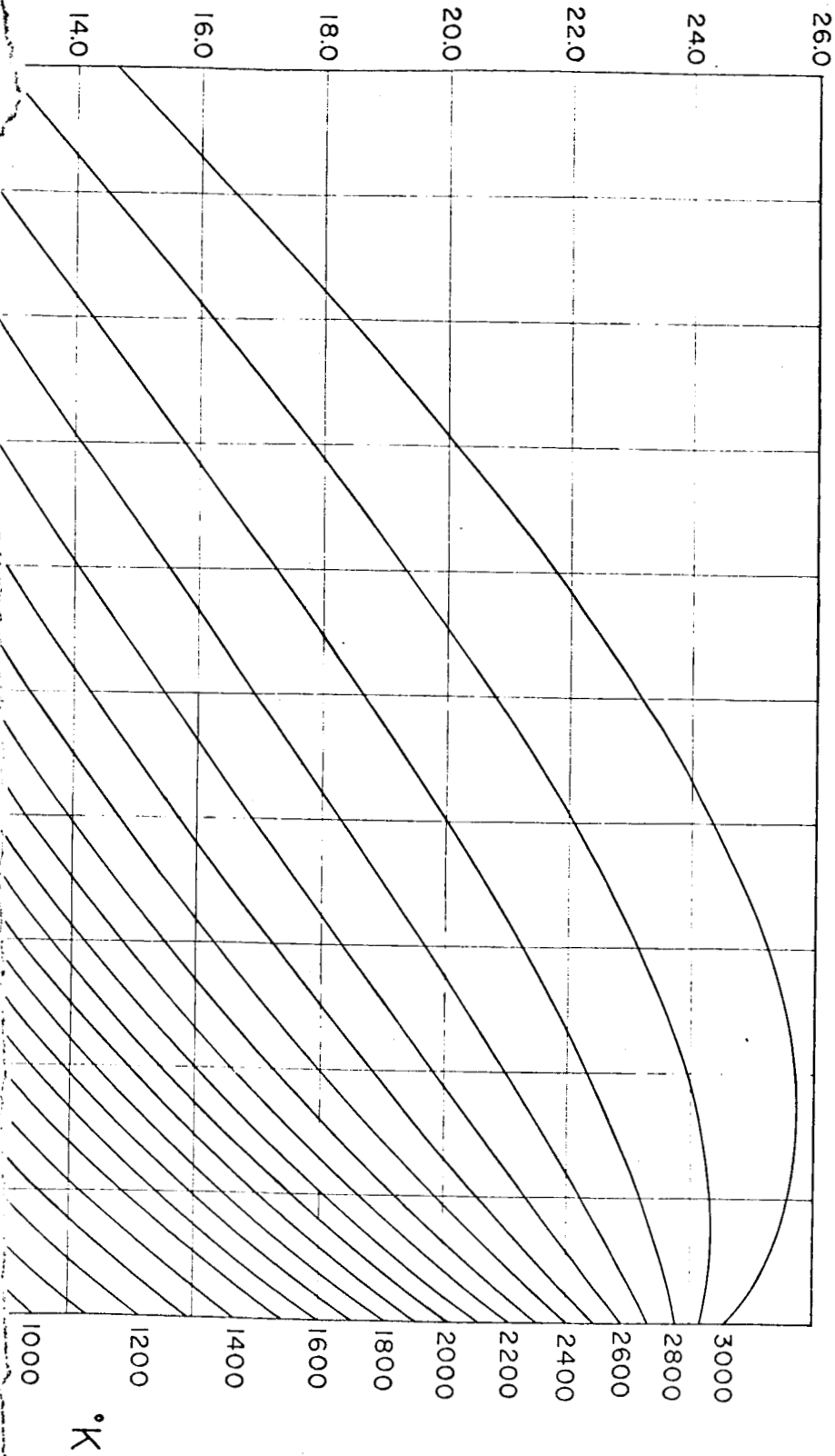


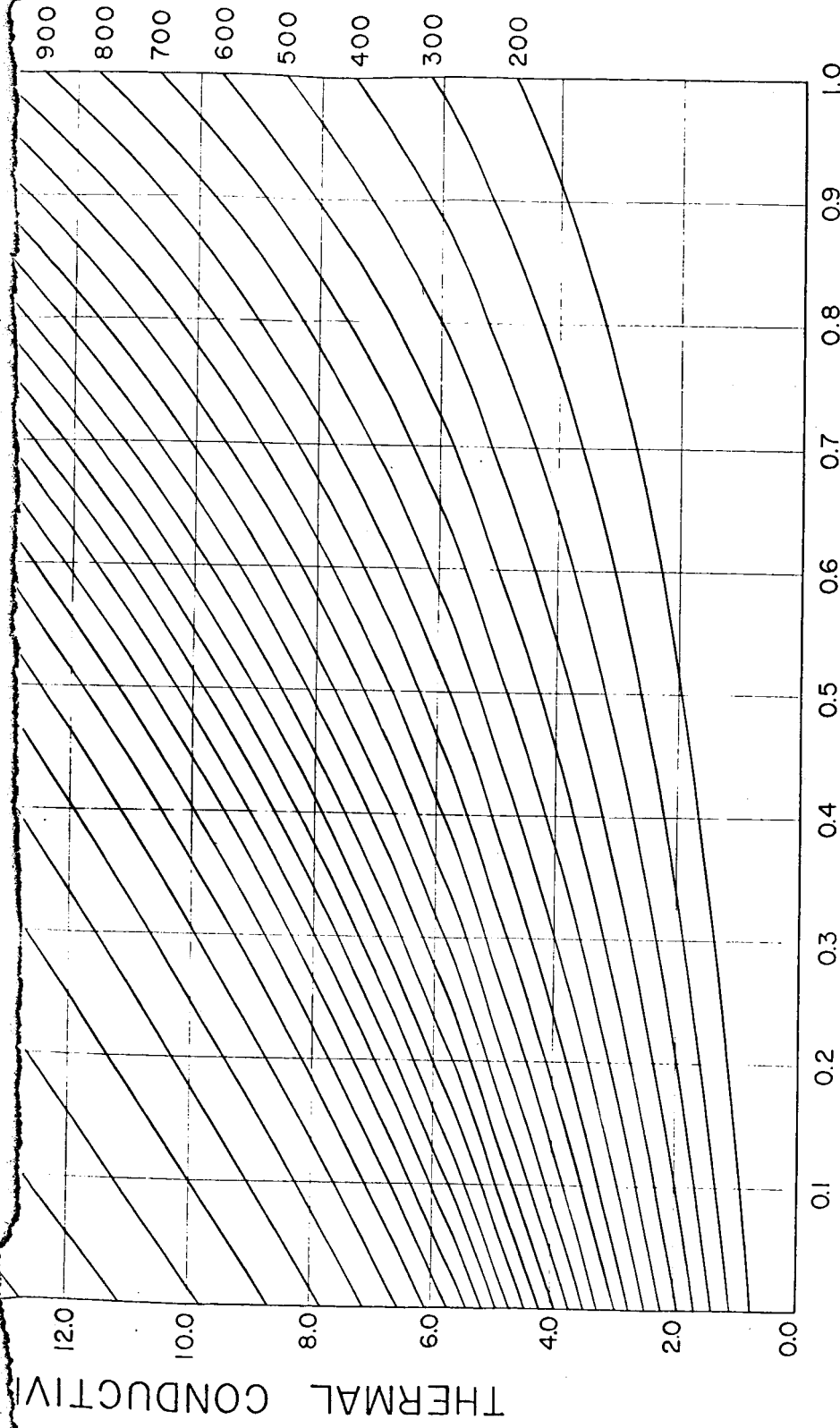


3A

1

TY RATIO --  $K/K_0$





HELIUM MOLE FRACTION --  $\left( \frac{\text{MOLE HE}}{\text{MOLE MIXTURE}} \right)$

3/B

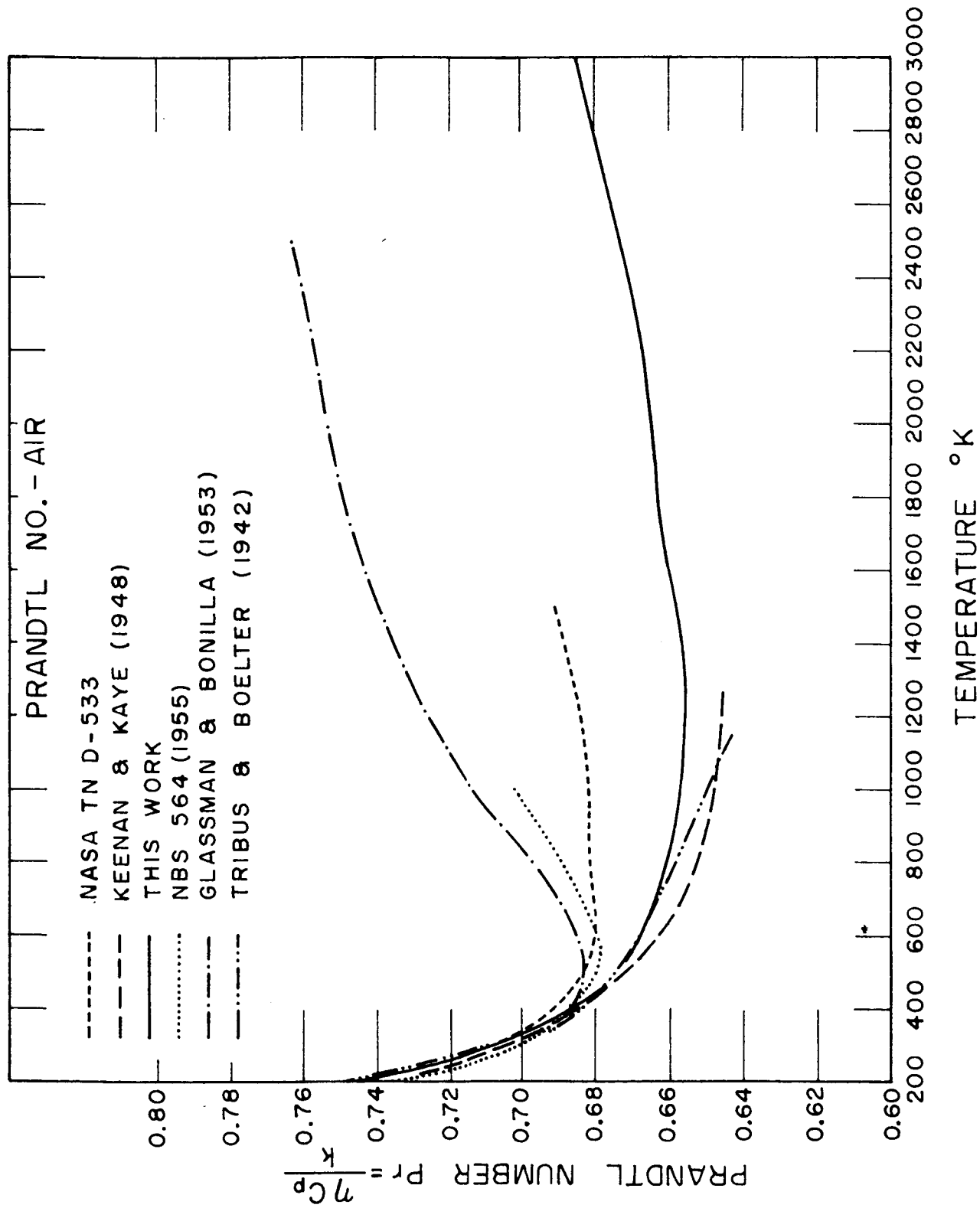


FIGURE 10